Math 280 Problems for November 6

Pythagoras Level

1. Find conditions on the parameters a, b, c, and d so that

$$f(x,y) = a\sin(x+y) + b\cos(x+y) + c\sin(x-y) + d\cos(x-y)$$

can be written as $f(x,y) = g(x)h(y)$.

2. Find positive integers n and $a_1, a_2, \ldots, a_n$ such that

$$a_1 + a_2 + \cdots + a_n = 2012$$

and the product $a_1 a_2 \cdots a_n$ is as large as possible.

Newton Level

3. If m and n are positive integers and $a < b$, find a formula for

$$\int_a^b \frac{(b-x)^m}{m!} \frac{(x-a)^n}{n!} dx$$

and use this to evaluate

$$\int_0^1 (1-x^2)^n dx.$$

4. Sum the series

$$\sum_{i=1}^{\infty} \frac{36i^2+1}{(36i^2-1)^2}.$$

Hint: $\displaystyle\sum_1^{\infty} \frac{1}{i^2} = \frac{\pi^2}{6}$.

Wiles Level

5. Let A, B and C be real square matrices of the same size, and suppose that A is invertible. Prove that if $(A-B)C = BA^{-1}$, then $C(A-B) = A^{-1}B$.

6. Let f be a real-valued function with $n+1$ derivatives at each point of $\mathbb{R}$. Show that for each pair of real numbers a, b, $a < b$, such that

$$\ln\left(\frac{f(b) + f'(b) + \cdots + f^{(n)}(b)}{f(a) + f'(a) + \cdots + f^{(n)}(a)}\right) = b - a$$

there is a number c in the open interval (a,b) for which

$$f^{(n+1)}(c) = f(c)$$