Math 280 Problems for September 25

Pythagoras Level

#1. Let x; y; z be real numbers such that

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{x+y+z}.$$

Prove that either $x+y=0$ or $y+z=0$ or $z+x=0$.

#2. Seven people come to a party, each with his/her own coat. At the end of the party, the coats are randomly distributed to the people, one coat per person. What is the average number, over all 7! possibilities, of the number of people who get their own coat? For example, if there were only two people at the party, there would only be two possibilities – either both people would get back their own coat or each would get the other person's coat. This gives an average of $(2+0)/2=1$, in this case.

Newton Level

#3.Let g be the function defined by

$$g(x)=\begin{cases}\frac{\sin(x)}{x} & x\neq 0\\ 1 & x=0\end{cases}$$

Let h be the function defined by $h(x)=\int_x^\pi g(t)dt$. Find the area of the region bounded by the curve $y=h(x)$, $x=0$, $x=\pi$, and the x-axis.

#4. Evaluate the following limit:

$$\lim_{n\to\infty}\frac{(2^3-1)(3^3-1)(4^3-1)\cdots(n^3-1)}{(2^3+1)(3^3+1)(4^3+1)\cdots(n^3+1)}.$$

Wiles Level

#5. All of the positive integers are printed, in order, on an infinite strip:

12345678910111213991001010102

Then, all zeroes are erased to give:

1234567891 111213991 1 11 2

Then all the spaces are removed and the tape is cut into 4-digit strips:

1234	5678	9111	1213

Show that every single 4-digit sequence, *abcd*, with a, b, c, $d\in\{1,2,...9\}$, appears on infinitely many strips.

#6. Let N_n denote the number of ordered n-tuples of positive integers $(a_1,a_2,\ldots,a_n)$ such that $1/a_1+1/a_2+\ldots+1/a_n=1$. Determine whether N_{10} is even or odd.