

# Math 280 Problems for September 3

## Pythagoras Level

#1. In popular culture, many of us are familiar with the stereotype of the mad scientist. In this case, a mad veterinarian invents an animal transmogrifying machine. The machine can transmogrify:

- Two cats into one cat, or vice-versa
- One cat and one dog into one dog, or vice-versa
- Two dogs into one cat, or vice-versa

Beginning with three cats and one dog, is it possible to end up with

- (a) one dog and no cats?
- (b) one cat and no dogs?

Be sure to justify your answers.

#2. If the  $p$ th term of an arithmetic progression is  $q$  and the  $q$ th term is  $p$ , where  $p \neq q$ , find the  $(p + q)$ th term.

## Newton Level

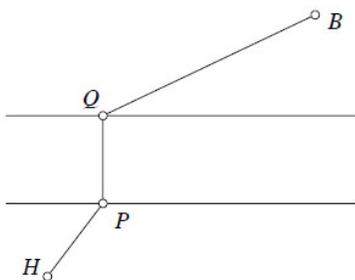
#3. Evaluate the integral

$$I = \int_{1/2}^2 \frac{\ln x}{1+x^2} dx.$$

#4. A smooth function  $f(x)$  has  $f''(x) > 0$  for all  $x$  in  $[0, 1]$ . For each point  $a$  in  $[0, 1]$ , draw the tangent line to  $y = f(x)$  at the point where  $x = a$ . Let  $A(a)$  be the area bounded by the curve  $y = f(x)$ , the tangent line at  $a$ ,  $x = 0$ , and  $x = 1$ . For what value of  $a$  is the area minimized?

## Wiles Level

#5. A farmer lives in a farmhouse  $H$  on one side of a stream bounded by two parallel lines. He often has to walk to his barn  $B$  on the other side of the stream. Since he is tired of getting wet, he wants to build a bridge  $PQ$  perpendicular to the stream, with  $P$  on the same side of the stream as  $H$ . He also wants the total walking distance  $HP + PQ + QB$  to be as short as possible. How should he determine where to place the bridge?



#6. How many rearrangements of the string of letters  $abcde$  have exactly two letters in their original places? The two  $a$ s are indistinguishable, so an  $a$  in either the first or second position is considered to be in its original place.