

Math 280 Problems for September 10

Pythagoras Level

#1. You have an integer with many digits. This integer is one more than a multiple of 9. One of the digits is removed and the remaining digits are rearranged. The new number is 8 more than a multiple of 9. What digit was removed? Justify your answer.

#2. How many ordered triples of integers (x, y, z) satisfy the equation $|x| + |y| + |z| = 2010$?

Newton Level

#3. Show that for every $0 < \theta \leq \pi$,

$$\int_0^\theta \sqrt{1 + \cos^2(t)} dt > \sqrt{\theta^2 + \sin^2(\theta)}.$$

#4. Find the area of the set of all points in the unit square, which are closer to the center of the square than to its sides.

Wiles Level

#5. Let f_n denote the n th Fibonacci number: $f_0 = 0$, $f_1 = 1$, and $f_n = f_{n-1} + f_{n-2}$ for $n \geq 2$. Fix $c \geq 2$. Evaluate

$$\sum_{n=1}^{\infty} \frac{f_n}{c^n}.$$

#6. A right circular cone has base of radius 1 and height 3. A cube is inscribed in the cone so that one face of the cube is contained in the base of the cone. What is the side-length of the cube?