Math 280 Problems for September 17

Pythagoras Level

#1. The following expression is a rational number. Which one?

$$4\sqrt{3} + \sqrt{129 - 72\sqrt{3}}.$$

#2. A teacher decides to hold a canned food drive, and asks each of his 29 students to bring in n cans. When the drive is over, the cans completely fill a certain number of boxes which hold exactly 72 cans each, with 3 cans left over. What is the smallest possible value for n?

Newton Level

#3. Suppose $f : \mathbb{R} \to \mathbb{R}$ is a continuous function which satisfies $f(-5) = 8$ and $f(0) = 2$, and is **even**. Define a new function g by

$$g(x) = \begin{cases} f(x) & \text{if } x \leq 0 \\ 4 - f(x) & \text{if } x > 0. \end{cases}$$

Compute

$$\int_{-5}^{5} g(x)\, dx.$$

#4. Compute the sum of the infinite series:

$$\sum_{n=0}^{\infty} \frac{(n+1)^2}{n!} = \frac{1^2}{0!} + \frac{2^2}{1!} + \frac{3^2}{2!} + \frac{4^2}{3!} + \cdots$$

Wiles Level

#5. A real-valued sequence is defined recursively by $a_0 = 5$ and $a_{n+1} = 6/(8 - a_n)$ for $n \geq 1$. Determine the limit of this sequence, or explain why the limit does not exist.

#6. After a lousy April Fool's day with not a single bite, a fisherman is extremely lucky for the remaining 29 days of the month, catching at least one fish each day. When he brags about this fact, and tells the total number of fish caught, a mathematician friend observes that there must have been a continuous stretch of days over which precisely 10 fish were caught. What is the maximum number of fish that the fisherman could have caught for the whole month?