Math 280 Problems for October 22

Pythagoras Level

Problem 1: Solve for x.

$$\sum_{i=0}^{2010} \binom{2010}{i} 4^{\frac{i}{2}} = x^{201}$$

Problem 2: Let $n \geq 1$. Pick at random a function

$$f : \{1, ..., n\} \to \{1, 2, 3\}$$

What is the probability Π of f not being onto (surjective)?

Newton Level

Problem 3: Evaluate the limit

$$\lim_{n \to \infty} \frac{1}{n}(e^{\frac{1}{n}} + e^{\frac{2}{n}} + e^{\frac{3}{n}} + \cdots + e^{\frac{n}{n}})$$

Problem 4: Find the power series (expanded about x = 0) for $\sqrt{\dfrac{1+x}{1-x}}$.

Wiles Level

Problem 5: Let $n \geq 2$ be an integer and define $f(x) = 1 - x^n$. For each $t \in (0, 1)$, let A_t denote the area of the triangle in the first quadrant formed by the x-axis, y-axis, and the tangent line to $f(x)$ at $x = t$. Find $t \in (0, 1)$ so that A_t is a minimum.

Problem 6: Let S be a set of real numbers which is closed under multiplication (that is, if a and b are in S, then so is ab). Let T and U be disjoint subsets of S whose union is S. Given that the product of any *three* (not necessarily distinct) elements of T is in T and that the product of any three elements of U is in U, show that at least one of the two subsets T, U is closed under multiplication.