MATH 440: Chapter 1 Write-Up Problems

Name:

1. For every pair of real numbers a and b, define a function $f_{a,b} : \mathbb{R} \to \mathbb{R}$ by the formula

$$f_{a,b}(x) = ax + b.$$

   (a) Show that $f_{1,b} \circ f_{a,0} = f_{a,b}$.

   (b) Prove or disprove that $f_{a,b}^{-1}$ exists. (Note: $f_{a,b}^{-1}$ satisfies $f_{a,b}^{-1} \circ f_{a,b} = f_{1,0}$.)

2. Let $M_2(\mathbb{R})$ be the 2×2 matrices with real entries. Set

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

and let

$$\mathcal{B} = \{X \in M_2(\mathbb{R}) \mid AX = XA\}.$$

   (a) Prove or disprove: if $P, Q \in \mathcal{B}$, then $P + Q \in \mathcal{B}$.

   (b) Prove or disprove: if $P, Q \in \mathcal{B}$, then $PQ \in \mathcal{B}$.

3. Let $f : X \to Y$ be an onto map of sets. For $a, b \in X$, consider the relation

$$a \sim b \text{ if and only if } f(a) = f(b).$$

Prove or disprove that this is an equivalence relation.