MATH 440: Chapter 2 Write-Up Problems

Name:

1. Prove or disprove: If $\gcd(a, r) = c$ and $\gcd(b, r) = d$, then $\gcd(ab, r) = cd$.
2. Give, with proof, a closed formula (i.e. a formula without "$\cdots$") for
$$1 + x + x^2 + x^3 + \cdots + x^{n-1} + \frac{x^n}{x-1}$$
for any $n \geq 1$.
3. Prove that if $a^n \mid b^n$ then $a \mid b$. (Hint: Set $d = \gcd(a, b)$ and write $a = rd$ and $b = sd$, where $\gcd(r, s) = 1$. Then you can use without proof that if $\gcd(r, s) = 1$, then $\gcd(r^n, s^n) = 1$. Then show that $r = 1$.)
4. Find, with proof, all integers x such that $3x + 7$ is divisible by 11.