MATH 440: Chapter 17 Write-Up Problems

Name:

Recall: When it's a disproof, you get an addition 2 bonus points for stating a true statement and proving it.

1. Let F be a field and let

$$I = \{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0 \mid a_i \in F \text{ and } a_n + a_{n-1} + \cdots + a_0 = 0\}.$$

   Show that I is an ideal of $F[x]$. By Theorem 17.12, it is principal, so also give the generator.

2. In the ring $\mathbb{R}[x]/\langle x^2+7x+2\rangle$, compute the multiplicative inverse of $3x+1$. In other words find $(3x+1)^{-1} \mod x^2+7x+2$.

3. Prove or disprove: The ideal $\langle x\rangle$ in $\mathbb{Q}[x]$ is maximal.