MATH 440: Chapters 6 and 9 Write-Up Problems

Name:

Recall: When it's a disproof, you get an addition 2 bonus points for stating a true statement and proving it.

1. Prove or disprove: If $g_1H = g_2H$, then $Hg_1 = Hg_2$.
2. Prove or disprove: $(\mathbb{R}, +)$ is isomorphic to $(\mathbb{R}^\times, \cdot)$.
3. Let $I_{\mathbb{R}^\times} = \left\{ \begin{pmatrix} r & 0 \\ 0 & r \end{pmatrix} \mid r \in \mathbb{R}^\times \right\}$. Then $I_{\mathbb{R}^\times} \subseteq \mathrm{GL}_2(\mathbb{R})$ and the set of right cosets

$$I_{\mathbb{R}^\times}\mathrm{GL}_2(\mathbb{R}) = \{I_{\mathbb{R}^\times}M \mid M \in \mathrm{GL}_2(\mathbb{R})\}$$

forms a group under matrix multiplication. Let the linear transformations on $\mathbb{C}$ be denoted

$$L = \left\{ f(x) = \frac{ax+b}{cx+d} \mid a, b, c, d \in \mathbb{R}, ad - bc \neq 0 \right\}.$$

Then L is a group under functional composition. Prove: $I_{\mathbb{R}^\times}\mathrm{GL}_2(\mathbb{R}) \cong L$.