WINONA STATE UNIVERSITY

COLLEGE OF SCIENCE AND ENGINEERING

DEPARTMENT OF MATHEMATICS AND STATISTICS

**Course Outline – MATH 242**

**Title:** Linear Algebra

**Number of Credits:** 4

**Catalog Description:**  An introduction to using linear algebra techniques and tools to solve and extract data from systems. Topics include: Systems of Linear Equations, Eigenvalues and Eigenvectors and SVD, Abstract vector spaces, Matrix invariants, Computer Algebra Systems (CAS). Prerequisite:  MATH 212 - Calculus I. Offered every semester.

**Possible Textbooks:**

* Linear Algebra by David Cherney, Tom Denton, Rohit Thomas, and Andrew Waldron (Note to Instructor: this text is very much oriented toward explaining the big picture of linear algebra techniques. It could be a base text if a second text is chosen for applications.
* Finite-Dimensional Linear Algebra by Mark Gockenbach (Note to instructor: The theory in this text is designed for an advanced linear algebra course, but certain sections have nice applications (e.g. SVD, best-fit, linear combinations, invariants of matrices)
* A text by
* Elementary Linear Algebra with Applications by Howard Anton (*Note to Instructor:* this text is very much oriented towards theory and therefore appropriate for the emphasis of this course, but possibly too much so. The text will work well if the instructor is prepared to offer supplemental explanation.)
* Linear Algebra and its Applications by David C. Lay (*Note to Instructor:* this text displays a nice balance between theory and application and is more accessible to our students than Anton. However, the general emphasis in this course is on theory.)
* Linear Algebra by Jim Hefferon (*Note to Instructor:* this text presents matrix theory and linear algebra techniques as a tool to study linear spaces. It contains both applications and theory.)

**Topics Covered:**

1. Introduction to Linear Systems
   1. Gaussian Operations
   2. Setting up and solving systems with CAS (coding terms, decoding results)
   3. Echelon Form and Reduced Echelon Form
   4. Matrix Multiplication
   5. Linear Vector Geometry in R^n
   6. Solution Sets of Linear Systems
   7. Applications: tournament ranking, iterative systems (Markov systems), Balancing equations, Leontief input-output examples
2. Vector Spaces
   1. Definition of a Vector Space (remove proof – emphasize closure under linear combinations, inverses, and zero)
   2. Examples of Vector Spaces (including function spaces, matrix spaces, complex spaces, or vector spaces over fields that will be used in the course )
   3. Norm and inner products (for R2, R3, and for function spaces, matrix spaces, and other spaces)
   4. Weights and Linear Combinations with applications to spaces other than R2, R3 vector spaces (e.g. images as matrices)
   5. Gram-Schmidt Process
   6. Applications: Linear approximation, Best-Fit systems (regression with polynomials, periodic functions, linear combinations with matrices (images) )
   7. Spanning Sets and Subspaces (de-emphasize theory)
   8. Linear Independence (de-emphasize theory)
   9. Basis and Dimension(de-emphasize theory)
3. Determinants
   1. Geometry of Determinants
   2. Properties of Determinants
   3. Laplace's Expansion
4. Eigenvalues, Eigenvectors, and Singular Values
   1. Eigenvalue/vector Application: Designing linear systems with eigenvalues and eigenvectors (including non-R^n examples)
   2. Eigenvalue/vector with CAS
   3. Geometry of eigenvalues and vectors
   4. SVD – generalization for non-square matrices
   5. SVD Application: image compression
   6. Determinant – Eigenvalue (or Singular Value) relationship
5. Matrix Block Structure
   1. Matrix multiplication as transformations between spaces or bases
   2. Interpretting block structure of matrices
6. Other topics may include (time permitting):
   1. Markov Chains
   2. Maps between Vector Spaces
   3. Rank – Nullity theorem

**Remarks:**

* Attempt should be made to have students work on ‘encoding’ and ‘decoding’ problems into CAS, and with the student deciding what CAS-provided procedure is appropriate for the problem. (To make linear algebra applicable and useful, we want students to work on knowing how to apply linear algebra to problems.)
* Attempts should be made to have students apply linear algebra and interpret the results to problems. (e.g. they don’t have to code finding RREF, eigenvalues, SVD – but they should be able to explain why they used the procedure and interpret the results.)
* Attempt should be made to have students explore mathematics objects in order to identify patterns, and to suggest conjectures. For example:
  + Identify relationships between linear combinations of columns of a matrix and type of solution, or number of solutions to the value of the determinant
  + Identify that determinant is product of eigenvalues (or relationship with singular values)
  + Identify role of weights in linear combinations, and subsequently role of larger singular values compared to smaller singular values
  + Conjecture about relative sizes of singular values for a data set by looking at vectors (e.g. look at images and predict relative sizes of singular values)
* Attempt should be made to have students justify claims or produce counter-examples throughout. For example:
  + Proving algebraic identities involving non-commutative multiplication.
  + Proofs involving the property of linearity (e.g. the differential operator, isometries, etc.)
  + Proving whether a given set of vectors is linear independent.
  + Many exercises in Lay such as the T/F questions in the supplementary exercises on p. 102 of Chapter 1.
* Instructors should emphasize how different mathematical “operations” perform the same function in different vector spaces. Examples can include:
  + Using integration or dot products for projecting one vector onto another
  + Using integration or dot products, or norms on matrices, for “norm”
  + Setting up best-fit for vector spaces with data (e.g. matrices), vectors, functions
  + Interpreting eigenvalues and eigenvectors for vectors in spaces other than Rn.
* Instructors are encouraged to use applications first, before theory, to motivate various topics and help students attach abstract ideas or thinking to concrete previously worked processes.

**Approximate pace of coverage:**

Varies, although one approach may be to devote 2-3 class periods per application followed by one lecture class period summarizing theory.

**Method of Instruction:** Methods may include group work, computer labs, discussion of examples, student presentations, and lecture.

**Evaluation Procedures:** Possible methods include examinations, quizzes, computer assignments, homework problems, and a final examination.

**Minnesota Transfer Curriculum:** Not Applicable

**MnSCU Learning Outcomes:**

* Students will demonstrate the ability to use matrices to solve linear systems.  Assessed within projects, homework, exams, and quizzes.
* Students will demonstrate the ability to interpret vectors and spaces geometrically.  Assessed within projects, homework, exams, and quizzes.
* Students will demonstrate an understanding of using matrices to solve application problems. Assessed within projects, homework, exams, and quizzes.
* Students will demonstrate an understanding of using linear algebra techniques including inner products, eigenvalues and eigenvectors, and Singular Value Decomposition to solve application problems.  Assessed within projects, homework, exams, and quizzes.

**Last Revised:** Fall 2017 by the Mathematics Subgroup