WINONA STATE UNIVERSITY

COLLEGE OF SCIENCE AND ENGINEERING

DEPARTMENT OF MATHEMATICS AND STATISTICS

**Course Outline-MATH 312**

**Course Title:** Multivariable Calculus

**Number of Credits**: 4

**Catalog Description:** Multivariable functions and vector functions are studied as the concepts of differential and integral calculus are generalized to surfaces and higher dimensions. Topics include vectors, parametric equations, cylindrical and spherical coordinates, partial and directional derivatives, multiple integrals, line and surface integrals, and the theorems of Green, Gauss, and Stokes. Prerequisite: MATH 213 - Calculus II.

**Possible Textbooks:** Calculus: Early Transcendentals, by James Stewart (7th ed.)

Calculus Volume 3 - Open Stax ([**https://openstax.org/subjects**](https://openstax.org/subjects)**)**

**Topics Covered:**

1. Vectors and the Geometry of Space
   1. Three-Dimensional Coordinate Systems
   2. Vectors
   3. The Dot Product
   4. The Cross Product
   5. Equations of Lines and Planes
   6. Cylinders and Quadric Surfaces
2. Vector Functions
   1. Vector Functions and Space Curves
   2. Derivatives and Integrals of Vector Functions
   3. Arc Length and Curvature
   4. Motion in Space: Velocity and Acceleration
3. Partial Derivatives
   1. Functions of Several Variables
   2. Limits and Continuity
   3. Partial Derivatives
   4. Tangent Planes and Linear Approximations
   5. The Chain Rule
   6. Directional Derivatives and the Gradient Vector
   7. Maximum and Minimum Values
   8. Lagrange Multipliers
4. Multiple Integrals
   1. Double Integrals over Rectangles
   2. Iterated Integrals
   3. Double Integrals over General Regions
   4. Double Integrals in Polar Coordinates
   5. Applications of Double Integrals
   6. Triple Integrals in Cartesian Coordinates
   7. Triple Integrals in Cylindrical Coordinates
   8. Triple Integrals in Spherical Coordinates
   9. Change of Coordinates and Jacobian Matrices
5. Vector Calculus
   1. Vector Fields
   2. Line Integrals
   3. The Fundamental Theorem for Line Integrals
   4. Green’s Theorem
   5. Curl and Divergence
   6. Parametric Surfaces and Their Areas
   7. Surface Integrals
   8. Stokes’ Theorem
   9. The Divergence Theorem
   10. Summary (optional)

**Listing of Sections to be Covered (Calculus: ET, 7th edition, by James Stewart):**

* **Chapter 12:** 1-6
* **Chapter 13:** 1-4
* **Chapter 14:** 1-8
* **Chapter 15:** 1-9
* **Chapter 16:** 1-9, with 10 optional.

**Remarks:**

* Some review of Conic Sections, Section 10.5, may be needed before covering Section 12.6 on Quadric Surfaces.
* Some review of Polar Coordinates, Section 10.3, may be needed before covering Section 15.4, Double Integrals in Polar Coordinates.

**Approximate Pace of Coverage:**

36 required sections in 14 weeks 🡪 approximately 2.8 sections per week.

**Method of Instruction:** Lecture-presentation, discussion, question-answer sessions,

use of calculators/computers, group work.

**Evaluation Procedure:** Homework, quizzes, projects, midterm exams, and a final exam.

# **General Education: Mathematics/Statistics Intensive:** *The following language should appear in the syllabus for this course.*

# This is a General Education course that satisfies the Mathematics/Statistics Intensive requirement. Mathematics 312 contains requirements and learning activities that promote students' abilities to...

1. practice the correct application of mathematical or statistical models that are appropriate to their prerequisite knowledge of those areas; and
2. make proper use of modern mathematical or statistical methods appropriate to their level of prerequisite knowledge, to include, if statistics is used in a substantive way, the use of a statistical package with graphics capability when appropriate.

Topics below which include such requirements and learning activities are indicated below using lowercase, boldface letters **a.-b.** corresponding to these requirements.

**Relation of Topics Covered to Mathematics/Statistics Intensive:**

* Differentiation
* Second-Order Partial Derivatives **a.-b.**
* Optimization
* Local Extrema **a.-b.**
* Unconstrained Optimization **a.-b.**
* Constrained Optimization and LaGrange Multipliers **a.-b.**
* Integration
* Definite Integrals in Higher Dimensions **a.-b.**
* Iterated Integrals **a.-b.**
* Triple Integrals **a.-b.**
* Double Integrals in Polar Coordinates **a.-b.**
* Integrals in Cylindrical and Spherical Coordinates **a.-b.**
* Other Topics
* Parameterized Curves (Motion, Velocity, and Acceleration) **a.-b.**
* Vector Fields and Flows **a.-b.**
* Line Integrals and Green's Theorem **a.-b.**
* Flux Integrals **a.-b.**
* Divergence Theorem, Curl, and Stoke's Theorem **a.-b.**

**Minnesota Transfer Curriculum:** Not Applicable

**MnSCU Learning Outcomes:**

* Students will demonstrate the ability to find the arc length and curvature of vector functions.
* Students will demonstrate the ability to use the partial derivatives to find directional derivatives and extreme values of functions.
* Students will demonstrate the ability to set up and evaluate multiple integrals in different coordinate systems.
* Students will demonstrate the ability to set up, find, and apply line integrals.
* Students will demonstrate the ability to use Green's Theorem.
* Students will demonstrate the ability to use Stokes' Theorem and the Divergence Theorem.

**Last Revised:** Fall 2017 by Felino G. Pascual