WINONA STATE UNIVERSITY

COLLEGE OF SCIENCE AND ENGINEERING

DEPARTMENT OF MATHEMATICS AND STATISTICS

**Course Outline-MATH 327**

**Course Title:** Foundations of Mathematics

**Number of Credits**: 4

**Catalog Description:** With an emphasis on mathematical proof writing, the following topics are covered: structure of the real and complex numbers, elementary number theory, introductory group and field properties, basic topology of the reals. Prerequisite: MATH 242 - Linear Algebra and MATH 247 - Discrete Mathematics. Grade or P/NC. Offered each semester.

**Possible Textbooks:**

* A Transition to Advanced Mathematics by Smith, Eggen, St. Andre
* The Foundations of Mathematics by Sibley
* Discrete Mathematics with Applications by Susanna Epp
* How to Read and Do Proofs: An Introduction to Mathematical Thought Processes by Daniel Solow

**Topics Covered (in no particular order):**

1. Logic of Compound Statements
   1. Logical Form and Logical Equivalence
   2. Conditional Statements
   3. Valid and Invalid Arguments
2. The Logic of Quantified Statements
   1. Predicates and Quantified Statements
   2. Quantified Statements with Connectives
   3. Statements with Multiple Quantifiers
   4. Arguments with Quantified Statements
3. Methods of Proof
   1. Direct Proof and Counterexample
   2. Proofs by Contradiction and Contraposition
   3. Proofs with Multiple Parts
   4. Proofs involving Quantifiers
   5. Mathematical Induction and the Well-Ordering Principle
4. Elementary Number Theory
   1. Properties of Integers and Rational Numbers
   2. Divisibility Properties, GCD, LCM
   3. Fundamental Theorem of Arithmetic
   4. Division Algorithm
   5. Modular Addition and Multiplication
5. Set Theory
   1. Basic Definitions of Set Theory
   2. Set Operations
   3. Indexed Families of Sets
   4. Principles of Counting (optional)
   5. Cardinality of Infinite Sets (optional)
6. Relations
   1. Basic Definitions of Relations
   2. Relation Operations
   3. Properties of Relations
   4. Equivalence Relations, Equivalence Classes
   5. Partitions (optional)
   6. Ordering Relations (optional)
7. Functions
   1. Functions Defined on General Sets
   2. Function Operations
   3. Properties of Functions
   4. Functions as Relations (optional)
   5. Cardinality with Applications (optional)
8. Introduction to Topics in Analysis (optional)
   1. Sequences
   2. Properties of the Reals
   3. Basic Point-Set Topology
9. Introduction to Topics in Algebra (optional)
   1. Binary Operations on Sets
   2. Basic Introduction to Groups
   3. Basic Introduction to Rings

**Remarks:**

* The emphasis, during all topics of the course, is learning how to write proofs. This involves significant feedback from the instructor, but also possibly from peers.
* In addition to creating proofs, students should also learn to critique given “proofs”.
* The pace of the course should take into account that students are often learning both the technical ideas and the pertinent proof techniques in parallel.

**Approximate pace of coverage:**

27 required bulleted topics in 14 weeks 🡪 approximately 2.0 sections per week.

**Method of Instruction:** Lecture-presentation, discussion, question-answer sessions, group work.

**Evaluation Procedure:** Homework, quizzes, projects, period-long exams, and a final exam*.*

**Minnesota Transfer Curriculum:** Not Applicable

**MnSCU Learning Outcomes:**

* Students will demonstrate mastery of prerequisite material.
* Students will perform logical computations and set manipulations.
* Students will demonstrate knowledge of pertinent definitions and theorems
* Students will reason deductively in a variety of forms.
* Students will communicate mathematics correctly, clearly, and concisely, often in the form of a written proof.

**Last Revised:** Spring 2016 by the Mathematics Subgroup