Continued Fractions & a p-Adic Euclidean Algorithm

Presented by:

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Outline

- Continued fractions and the Euclidean Algorithm
- p-Adic numbers
- Continued fractions and the Euclidean algorithm in the p-adics

A Simple Continued Fraction is a fraction of the form:

$$a_o + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \ldots}}}$$

where a_0 is some integer and all other a_i's are positive integers

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This can be expressed as [2;5,2,4,3]

Euclidean Algorithm

The Euclidean Algorithm is used to determine the GCD of two integers a,b and is applied as follows:

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$$\vdots$$

$$\Rightarrow \quad r_{n-1} = r_n q_{n+1} + 0$$

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p-adic Number

Definition: A p-adic Number is a power series in the prime p.

There is a unique p-adic expansion for every real x,

$$x = \sum_{j=m}^{\infty} c_j p^j = c_m p^m + c_{m+1} p^{m+1} + c_{m+2} p^{m+3} + \ldots$$

where m is an integer, c_j are integers mod p.

Examples

$$3 = 3$$

$$4 + 5 \cdot 7 = 39$$

$$2 + 3 \cdot 7 + 1 \cdot 7^2 = 72$$

$$5 \cdot 7^{-1} + 2 = \frac{19}{7}$$

Example

What about:

$$4 + 3 \cdot 7 + 3 \cdot 7^2 + 3 \cdot 7^3 + 3 \cdot 7^4 + \cdots$$

Example

$$4 + 3\cdot 7 + 3\cdot 7^2 + 3\cdot 7^3 + 3\cdot 7^4 + \cdots$$
$$= 1 + (3\cdot 7^0 + 3\cdot 7^1 + 3\cdot 7^2 + 3\cdot 7^3 + 3\cdot 7^4 + \cdots)$$

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$$= 1 + 3(7^0 + 7^1 + 7^2 + \cdots)$$

Example

$$4+3\cdot 7+3\cdot 7^2+3\cdot 7^3+3\cdot 7^4+\cdots$$

$$=1+(3\cdot 7^0+3\cdot 7^1+3\cdot 7^2+3\cdot 7^3+3\cdot 7^4+\cdots)$$

$$=1+3(7^0+7^1+7^2+\cdots)$$

$$=1+3\left(\frac{1}{1-7}\right)$$

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$$= 1 + -\frac{3}{6}$$

$$= \frac{1}{2}$$

p-Adic Norm (Hensel 1897)

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Example: $\frac{63}{550}$ can be written as a product of primes as follows:

$$\frac{63}{550} = 2^{-1} \cdot 3^{2} \cdot 5^{-2} \cdot 7^{1} \cdot 11^{-1}$$

using the formula we have:

$$\left|\frac{63}{550}\right|_3 = 3^{-2} = \frac{1}{9}$$

The more positive powers of p, the “smaller” the number is!!

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Small is big/Big is small?

Compare 49 to 5/343

$$49 = 7^2 \qquad \frac{5}{343} = 5^1 \cdot 7^{-3}$$

Now use the 7-adic norm to find:

$$|49|_7 = 7^{-2} = \frac{1}{49} \qquad \left|\frac{5}{343}\right|_7 = 7^{-(-3)} = 343$$

so now

$$7^0 + 7^1 + 7^2 + \ldots$$ converges

Browkin's Model of Continued Fractions in p-adics (1978)

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We use the same type of method of pulling off the large portions and inverting the small.

$$\zeta_0 = b_0 + \cfrac{1}{b_1 + \cfrac{1}{b_2 + \cfrac{1}{\ddots + \cfrac{1}{\zeta_n}}}}$$

Where $\zeta_n = (\zeta_{n-1} - b_{n-1})^{-1}$ and b_{n-1} is the "big part" of ζ_{n-1}

$$\zeta_0 = \frac{69}{5} = 5 + 3 \cdot 11 + 2 \cdot 11^2 + 2 \cdot 11^3 + 2 \cdot 11^4 + \ldots$$

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$$\zeta_1 = \left(3\cdot 11 + 2\cdot 11^2 + 2\cdot 11^3 + 2\cdot 11^4 + \ldots\right)^{-1}$$

$$= \frac{4}{11} - 3 + 3\cdot 11 - 3\cdot 11^2 + 3\cdot 11^3 - \ldots$$

$$b_1 = \frac{4}{11} - 3 = \frac{-29}{11}$$

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$$b_1 = \frac{4}{11} - 3 = \frac{-29}{11}$$

$$\zeta_2 = \left(3\cdot 11 - 3\cdot 11^2 + 3\cdot 11^3 - \ldots\right)^{-1}$$

$$= \frac{4}{11}$$

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$$\zeta_2 = \left(3\cdot 11 - 3\cdot 11^2 + 3\cdot 11^3 - \ldots\right)^{-1}$$

$$= \frac{4}{11}$$

$$b_2 = \frac{4}{11}$$

$$\Rightarrow \frac{69}{5} = \left[5; \frac{-29}{11}, \frac{4}{11}\right]$$

$$\frac{69}{5} = 5 + \cfrac{1}{\cfrac{-29}{11} + \cfrac{1}{\cfrac{4}{11}}}$$

Computing p-adic inverses is hard

Example: Suppose we needed to compute

$$\left(-2\cdot 7-3\cdot 7^{2}+1\cdot 7^{3}+3\cdot 7^{4}-2\cdot 7^{5}-3\cdot 7^{6}+1\cdot 7^{7}+3\cdot 7^{8}-\ldots\right)^{-1}$$

The inverse does not start repeating until 420 terms in!

Browkin's method is similar to finding the simple continued fractions of reals. Can we use the Euclidean Algorithm instead?

Example, 69/5 Again

$69 = q \cdot 5 + 11r$

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$$69 = 5 \cdot 5 + 11 \cdot 4$$

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$$5 = q \cdot 11 \cdot 4 + 11^2 r$$

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$$\frac{69}{5} = \left[5, \frac{-29}{11}, \frac{4}{11}\right]$$

In general, $\frac{a}{b}$

$$a = q_1 \cdot b + p \cdot k_1$$

$$q_1 = ab^{-1} \bmod p$$

$$b = q_2 \cdot pk_1 + p^2 k_2$$

$$q_2 = \frac{bk_1^{-1} \bmod p^2}{p}$$

$$pk_1 = q_3 \cdot p^2 k_2 + p^3 k_3$$

$$q_3 = \frac{k_1 k_2^{-1} \bmod p^2}{p}$$

$$until \quad k_n = 0$$

Then, $\frac{a}{b} = [q_0; q_1, q_2, \ldots, q_n]$

Prove: $Our(q_i) = Browkin(b_i)$

We know:

$$y_i = r_{i-1}$$

$$x_i = y_{i-1}$$

$$x_i = q_i y_i + r_i$$

$$\zeta_i = (\zeta_{i-1} - b_{i-1})^{-1}$$

$$\Rightarrow b_i = \zeta_i - (\zeta_{i+1})^{-1}$$

$$\zeta_i = \frac{r_{i-2}}{r_{i-1}} \Rightarrow \zeta_{i+1} = \frac{r_{i-1}}{r_i}$$

Proof:

$$r_{i-2} = q_i r_{i-1} + r_i$$

$$q_i = \frac{r_{i-2} - r_i}{r_{i-1}} = \frac{r_{i-2}}{r_{i-1}} - \frac{r_i}{r_{i-1}}$$

$$\therefore q_i = \zeta_i - (\zeta_{i+1})^{-1} = b_i$$

Summary

•The Euclidean Algorithm is used to construct simple continued fractions

•Defined a p-adic number

•Browkin's Model for finding continued fractions in p-adics

•The new p-Adic Euclidean Algorithm for constructing continued fractions in p-adics

•The two methods are mathematically the same, but computationally our way is easier and faster.

Questions?

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