A p-adic Euclidean Algorithm

Cortney Lager
Winona State University

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Definitions

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$$\left|\frac{a}{b}\right|_p = p^{v(b)-v(a)}$$

where $v(n)$ is the number of times p divides the integer n.

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- Which is "smaller" 162 or $\frac{5}{27}$ according to the norm in the 3-adics?
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$$|162|_3 = 3^{-4} < \left|\frac{5}{27}\right|_3 = 3^3.$$
- Thus, 162 is "smaller" than $\frac{5}{27}$ in the 3-adics.

Definition

- An element $\zeta \in \mathbb{Q}_p$ is a power series in the prime p,

$$\zeta = \sum_{j=m}^{\infty} c_j p^j = c_m p^m + c_{m+1} p^{m+1} + c_{m+2} p^{m+2} + \dots$$

where m is a (possibly negative) integer and $c_j \in \left\{ \frac{1-p}{2}, \dots, \frac{p-1}{2} \right\}$.

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p-adic Divison Algorithm

Theorem

Given any s and $t \in \mathbb{Z}$, there exists uniquely $q \in \mathbb{Z}$ and $r \in \mathbb{Z}$ with $0 < r < t$ such that

$$s = qt + r.$$

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Example (in the 7-adics)

$$\frac{181625}{11} = \left(\frac{10555}{2}\right)$$

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$$\frac{181625}{11} = (2)\left(\frac{10555}{2}\right) + \left(\frac{9360}{11} \cdot 7^1\right)$$

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This process either continues indefinitely or stops when $\eta_i = 0$. The outputs of this algorithm are the sequences $\{q_i\}$ and $\{\eta_i\}$.

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Greatest Common Divisor

Definition

Let a, b $\in \mathbb{Z}$, then the g=(a,b) is the positive integer that satisfies the following properties,

(i.) $\frac{a}{g}, \frac{b}{g} \in \mathbb{Z}$ and,

(ii.) if there exists f with $\frac{a}{f}, \frac{b}{f} \in \mathbb{Z}$, then $\frac{g}{f} \in \mathbb{Z}$.

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We can extend the definition of the gcd to $\frac{a}{b}, \frac{c}{d} \in \mathbb{Q}$ by setting

$$\left(\frac{a}{b}, \frac{c}{d}\right) = \frac{\gcd(a,c)}{\operatorname{lcm}(b,d)}.$$

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Theorem

Let $\sigma, \tau \in \mathbb{Q} \subseteq \mathbb{Q}_p$ with $\sigma = sp^{v(\sigma)}$, $\tau = tp^{v(\tau)}$. Then the p-adic Euclidean Algorithm applied to σ and τ stops after k-steps and if $\eta_i = h_i p^{\epsilon_i}$, then $|h_{k-1}| = \gcd(t, s)$.

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$$\Rightarrow \gcd\left(\frac{181625}{11}, \frac{10555}{2}\right) = \frac{5}{22}.$$

Classical Simple Continued Fraction

Definition

$$\frac{a}{b} = q_1 + \cfrac{1}{q_2 + \cfrac{1}{q_3 + \cfrac{1}{\ddots}}}$$

where q_i are positive integers.

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Theorem

The q_is are the quotients from the Euclidean Algorithm of a and b.

p-adic Simple Continued Fraction

Definition (**Browkin**)

For $\zeta \in \mathbb{Q}_p$,

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Browkin's method computes the b_is through a series of p-adic inverses.

p-adic Simple Continued Fraction

Example

$$\frac{72650}{23221} = 2 + \cfrac{1}{\frac{12}{7} + \cfrac{1}{\frac{-10}{7} + \cfrac{1}{\frac{50}{7^2} + \cfrac{1}{\frac{2}{7}}}}}$$

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Theorem

Let $\{q_i\}$ be the outputs of the p-adic Euclidean Algorithm applied to σ and τ, then $\frac{\sigma}{\tau} = q_1 + \cfrac{1}{q_2 + \cfrac{1}{q_3 + \cfrac{1}{\ddots}}}$

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- Finally, I would like to give a special thank you to Dr. Eric Errthum for begin my mentor throughout this project.

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- **Questions?**