Math 280 Problems for September 1?

Pythagoras Level

Problem 1: A confused bank teller transposed the dollars and cents when he cashed a check for Ms Smith, giving her dollars instead of cents and cents instead of dollars. After buying a newspaper for 50 cents, Ms Smith noticed that she had left exactly three times as much as the original check. What was the amount of the check? (Note: 1 dollar = 100 cents.)

Problem 2: Five men crash-land their airplane on a deserted island in the South Pacific. On their first day they gather as many coconuts as they can find into one big pile. They decide that, since it is getting dark, they will wait until the next day to divide the coconuts. That night each man took a turn watching for rescue searchers while the others slept. The first watcher got bored so he decided to divide the coconuts into five equal piles. When he did this, he found he had one remaining coconut. He gave this coconut to a monkey, took one of the piles, and hid it for himself. Then he jumbled up the four other piles into one big pile again.

To cut a long story short, each of the five men ended up doing exactly the same thing. They each divided the coconuts into five equal piles and had one extra coconut left over, which they gave to the monkey. They each took one of the five piles and hid those coconuts. They each came back and jumbled up the remaining four piles into one big pile.

What is the smallest number of coconuts there could have been in the original pile?

Newton Level

Problem 3: For real x, let $< x >$ denote the fractional part of x. Thus, $< x >$ = x - $\lfloor x \rfloor$, where $\lfloor x \rfloor$ is the greatest integer less than or equal to x. Evaluate

$$\int_{-1}^{1} < x^2 + 2x - 3 > dx.$$

Problem 4: Show that

$$\sum_{n=1}^{\infty} \frac{n}{3 \cdot 5 \cdot 7 \cdots (2n+1)} = \frac{1}{2}.$$

Wiles Level

Problem 5: Find the volume of the region of points (x, y, z) such that

$$(x^2 + y^2 + z^2 + 8)^2 \le 36(x^2 + y^2).$$

Problem 6: For each continuous function $f : [0, 1] \to \mathbb{R}$, let $I(f) = \int_0^1 x^2 f(x)\, dx$ and $J(x) = \int_0^1 x\, (f(x))^2\, dx$. Find the maximum value of $I(f) - J(f)$ over all such functions f.