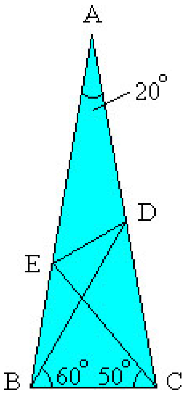
Math 280 Problems for September 20

Pythagoras Level

Problem 1: How many of the integers from 1 to 2008 may be written as the sum of two or more distinct integral powers of 3? (For example, $28 = 3^0 + 3^3$ is such an integer.) Justify your answer.

Problem 2: Let ABC be an isosceles triangle ($AB = AC$) with $\angle BAC = 20°$. Point D is on side AC such that $\angle DBC = 60°$. Point E is on side AB such that $\angle ECB = 50°$. Find, with proof, the measure of $\angle EDB$.



Newton Level

Problem 3: Find the value of the infinite product

$$P = \frac{7}{9} \times \frac{26}{28} \times \frac{63}{65} \times \cdots \times \frac{k^3-1}{k^3+1} \times \cdots$$

Problem 4: Find all real solutions (x, y) of the system

$$\begin{aligned} |x| + x + y &= 10, \\ x + |y| - y &= 12. \end{aligned}$$

Justify your answer.

Wiles Level

Problem 5: Consider a set S with binary operation $\circledast$, i.e. for each $a, b \in S$, $a \circledast b \in S$. Assume $(a \circledast b) \circledast a = b$ for all $a, b \in S$. Prove that $a \circledast (b \circledast a) = b$ for all $a, b \in S$. Note: We do not know in general if $a \circledast b = b \circledast a$ (commutivity) or if $a \circledast (b \circledast c) = (a \circledast b) \circledast c$ (associativity).

Problem 6: Let f be a real function satisfying $f(x) + y = f(x + y)$ for all real x and y. Assume that $f(0)$ is a positive integer, and that $f(2) \mid f(5)$. Find $f(2008)$. Note: For integers m and n, the symbol $m \mid n$ means that m divides n.