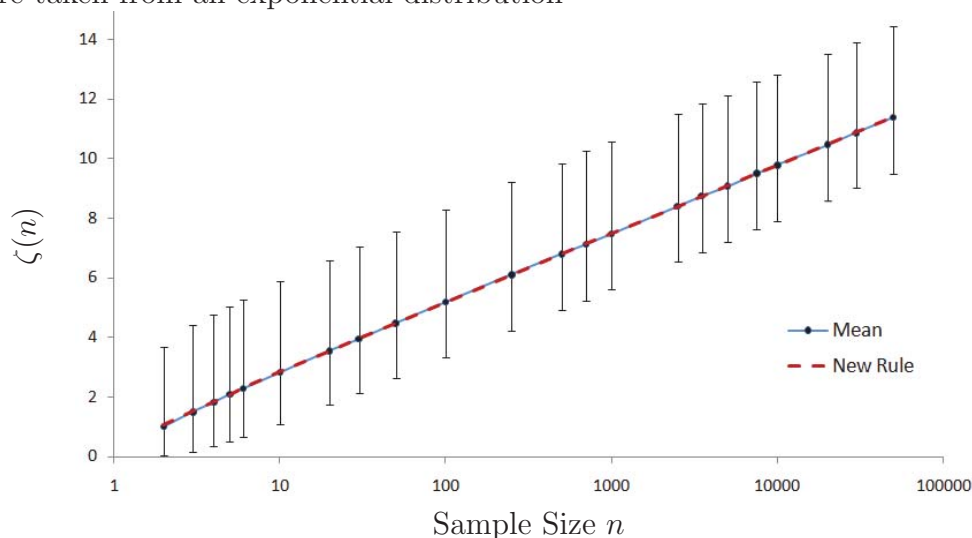


Figure 5: Mean, 2.5% and 97.5% percentiles of  $\zeta$  by the size of samples which were taken from an exponential distribution



**Empirical Rule of Thumb 5.3.** For samples of size  $n$  from an exponential distribution:

$$\sigma \approx \frac{\text{first quasi-range}}{\ln(n-2) + (\gamma-1)}.$$

### 5.3 Normal Distribution

The standard normal distribution is defined by:

$$p(x) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-x^2}{2}\right),$$

so

$$P(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-w^2/2} dw.$$

To analyze the normal distribution as we did the other distributions above we would need to integrate  $wf(w)$  using the p.d.f.  $f(w)$  from Equation 5.1. Finding  $f(w)$  alone requires us to integrate powers of the integral for  $P(x)$  above. At the time of writing this paper we are still working with Maple to try and approximate these values of  $E(W_r)$  using Maple's built-in error function.

## 6 Conclusion

In this research project we sought to understand the naive rule of thumb  $\sigma = \text{range}/4$ . It is now clear that a useful rule of thumb must include information from both the sample size

and the underlying distribution.

In the future it would be interesting to study rules based on the quasi-ranges (such as our rules of thumb 5.1 and 5.3) and especially those made from linear combinations of quasi-ranges. These should be less sensitive to outliers. We also would like to prove the conjectured results of section 5.2 and succeed in theoretically approximating expected values for the quasi-ranges of the normal distribution.

We also considered exploring rules of thumb for the Gamma and Beta distributions, but will leave these to the reader.

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