

# CONDITIONS FOR ROBUST PRINCIPAL COMPONENT ANALYSIS

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**Abstract.** Principal Component Analysis (PCA) is the problem of finding a low-rank approximation to a matrix. It is a central problem in statistics, but it is sensitive to sparse errors with large magnitudes. Robust PCA addresses this problem by decomposing a matrix into the sum of a low-rank matrix and a sparse matrix, thereby separating out the sparse errors. This paper provides a background in robust PCA and investigates the conditions under which an optimization problem, Principal Component Pursuit (PCP), solves the robust PCA problem. Before introducing robust PCA, we discuss a related problem, sparse signal recovery (SSR), the problem of finding the sparsest solution to an underdetermined system of linear equations. The concepts used to solve SSR are analogous to the concepts used to solve robust PCA, so presenting the SSR problem gives insight into robust PCA. After analyzing robust PCA, we present the results of numerical experiments that test whether PCP can solve the robust PCA problem even if previously proven sufficient conditions are violated.

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