# Math 280 Problems for September 18

## Pythagoras Level

#1. (a) Show that the sum
\[
1 + 2 + 3 + \cdots + 2009 + 2010 + 2009 + \cdots + 3 + 2 + 1
\]

is the square of an integer.

(b) Generalize the result in (a), with proof.

#2. A game board consists of a linear path of 2010 squares, numbered from 1 to 2010. A game piece is initially on square 1, and two players alternately move it. On each move a player advances the piece by 1, 2, 3, 4, 5 or 6 squares. Thus, the first player advances the piece to square 2, 3, 4, 5, 6 or 7. The player who moves onto square 2010 wins. Describe a winning strategy for one of the players, and make clear that it wins.

## Newton Level

#3. Suppose that the function \( f \) satisfies \( f'(x) = 1 + f(x) \) for all \( x \). If \( f(2) = 3 \), find:
(a) \( f^{(10)}(2) \) (where \( f^{(10)} \) denotes the 10th derivative of \( f \));
(b) \( f(3) \).

Justify your answers.

#4. Evaluate
\[
\lim_{x \to \infty} \int_x^{2x} \frac{dt}{\sqrt{t^3 + 4}}
\]

and justify your answer.

## Wiles Level

#5. Prove that for every rational number \( a \), the equation
\[
y = \sqrt{x^2 + a}
\]

has infinitely many solutions \((x, y)\) with \( x \) and \( y \) rational.

#6. For a partition \( \pi \) of \( \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \), let \( \pi(x) \) be the number of elements in the part containing \( x \). Prove that for any two partitions \( \pi \) and \( \pi' \), there are two distinct numbers \( x \) and \( y \) in \( \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \) such that \( \pi(x) = \pi(y) \) and \( \pi'(x) = \pi'(y) \). [A partition of a set \( S \) is a collection of disjoint subsets (parts) whose union is \( S \).]