Math 280 Problems for October 23

Pythagoras Level

1. Let $a_1 = 3$ and for $n \geq 1$, $a_{n+1} = a_n^2 - 2$. Prove that if $m \neq n$ then a_m and a_n are relatively prime.

2. Let $f_1(x) = f(x) = \frac{1}{1-x}$, and for $n > 1$, $f_n(x) = f(f_{n-1}(x))$. Evaluate $f_{2011}(2010)$.

Newton Level

3. Find the maximum and minimum values of

$$2x|x| - 5x + 1,$$

for $|x + 1| \leq 3$. Justify your answer.

4. Evaluate

$$\int_1^2 \frac{1}{\lfloor x^2 \rfloor} dx,$$

where as usual $\lfloor u \rfloor$ denotes the greatest integer less than or equal to u.

Wiles Level

5. If

$$x = \frac{1 + \sqrt{2010}}{2},$$

what is the value of

$$(4x^3 - 2013x - 2010)^{2015}?$$

Justify your answer.

6. Given that a, b and c are real numbers with $a < b$ and $a < c$, prove that

$$a < \frac{bc - a^2}{b + c - 2a} < \min\{b, c\}.$$