Math 280 Problems for October 30

Pythagoras Level

1. Two zombies randomly pop out of the ground along a straight line of length 2 meters. What is the probability they will be within $1/3$ meter apart?

2. You've been killing zombies all day and your genius side-kick just solved a differential equation and found that soon the amount of zombies left will be:

$$z = \sqrt[3]{2+\sqrt{5}} + \sqrt[3]{2-\sqrt{5}}.$$

However, she and her work are eaten by a zombie before she could simplify. Show that $z = 1$. (Note: All your computers and calculators were destroyed by zombies.)

Newton Level

3. A zombie is standing in a coordinate plane at $(1, 0)$. Your zombie death ray works best at a distance 1 from a zombie. You decide to run along a path given by $y = x^p$ from the point $(0, 0)$ to $(1, 1)$. For what positive real numbers p is the maximal distance from the zombie to your path equal to 1?

4. For p and q real number with $p > q$, compute

$$\int_0^1 (1 - x^{1/p})^q dx - \int_0^1 (1 - x^{1/q})^p dx$$

Hint: Zombies!

Wiles Level

5. For real numbers u, let $\{u\} = u - \lfloor u \rfloor$ denote the fractional part of u. Here $\lfloor u \rfloor$ denotes, as usual, the greatest integer less than or equal to u. For example, $\{\pi\} = \pi - 3$, and $\{-2.4\} = -2.4 - (-3) = 0.6$. Find all real x such that

$$\{(x+1)^3\} = x^3$$

6. Let G be the set of all continuous functions $f : \mathbb{R} \to \mathbb{R}$, satisfying the following properties.

   - $f(x) = f(x+1)$ for all x,
   - $\int_0^1 f(x)dx = 2010$.

   Show that there is a number α such that $\alpha = \int_0^1 \int_0^x f(x+y)dydx$ for all $f \in G$.