Math 280 Problems for October 29

Pythagoras Level

#1. Seven dead bodies, tattoed 1 through 7, are correctly placed in corresponding coffins (one to a box), also engraved 1 through 7. The dead rise again and crawl out of their graves. They are beaten back by townspeople with pitchforks and torches. The dead randomly returned to the coffins, one zombie to a box. What is the probability that no zombie will find its correct grave?

#2. The numbers $\pm 1, \pm 2, \ldots \pm 2010$ are written on a the back of 4020 zombies. You can use your shotgun to take out two zombies labeled x and y at a time, but each time you do this, a new zombie is spawned labeled with the product xy.You continue this process until only one zombie remains. Prove that the last zombie is positive.

Newton Level

#3. A zombie stands at one corner of a square cornfield and wishes to reach the diagonally opposite corner so she can eat your brains. If w is the zombie's walking speed and s is the zombie's crawling speed through corn ($s < w$), find the zombie's path for shortest time. [Consider two cases: (i) $w/s < \sqrt{2}$, and (ii) $w/s > \sqrt{2}$.]

#4. A zombie is chained to a North-South fence at a Zombie Amusement Park. Visitors are allowed to walk on a East-West running sidewalk (on the East side of the North-South Fence). There is a parabolic-shaped silo... oh screw it: Find the point in the first quadrant on the graph of $y = 7 - x^2$ such that the distance between the x- and y- intercepts of the tangent line at the point is minimum.

Wiles Level

#5. Suppose f is a continuous real-valued function on the interval $[0, 1]$. Show that

$$\int_0^1 x^2 f(x)\, dx = \frac{1}{3} f(\xi)$$

for some $\xi \in [0, 1]$.

#6. Suppose that

$$\frac{2x + 3}{x^2 - 2x + 2}$$

has the Taylor series

$$\sum_{k=0}^{\infty} a_k x^k.$$

Find the sum of the odd numbered coefficients, i.e., find

$$\sum_{k=0}^{\infty} a_{2k+1} = a_1 + a_3 + a_5 + \cdots .$$