WINONA STATE UNIVERSITY

COLLEGE OF SCIENCE AND ENGINEERING

DEPARTMENT OF MATHEMATICS AND STATISTICS

**Course Outline-MATH 452**

**Title:** Advanced Calculus I

**Number of Credits:** 3

**Catalog Description:** A systematic approach to the theory of differential and integral calculus for functions and transformations in several variables. Prerequisites: MATH 327 – Foundations of Mathematics and MATH 312 – Multivariable Calculus. Offered every Fall semester.

**Possible Textbooks:** Fundamental Ideas of Analysis by Michael Reed.

**Topics Covered:** Numbers in square brackets [ ] indicate the approximate number of class hours that should be spent on the topic.

1. Preliminaries
   1. The Real Numbers [1]
   2. Sets and Functions [1]
   3. Cardinality [1]
   4. Methods of Proof [2]
2. Sequences
   1. Convergence [2]
   2. Limit Theorems [2]
   3. Cauchy Sequences [2]
   4. Supremum and Infimum and the Bolzano-Weierstrass Theorem [6]
3. The Riemann Integral
   1. Continuity [2]
   2. Continuous Functions on Closed Intervals [2]
   3. The Riemann Integral [2]
   4. Numerical Methods (lightly)
   5. Discontinuities [2]
   6. Improper Integrals [3]
4. Differentiation
   1. Differentiable Functions [2]
   2. The Fundamental Theorem of Calculus [2]
   3. Taylor’s Theorem [2]
   4. Newton’s Method (lightly)
   5. Inverse Functions [2]

**Listing of Sections to be Covered in Reed:**

* **Chapter 1:** 1-4.
* **Chapter 2:** 1-2, 4-6.
* **Chapter 3:** 1-3, 5-6 with 4 optional.
* **Chapter 4:** 1-3, 5 with 4 optional.

**Remarks:**

* It is understood that the topological space in which all topics in the course reside is Rn, and topics in topology should be covered only insomuch as they are needed to provide a context for the proofs in Rn. Instructors are encouraged to motivate the course with some classic pathological examples, whose analysis necessitated the development of rigorous methods of proof. However, the main thrust of the course is for students to develop and demonstrate the ability to read, write, and understand rigorous mathematical arguments relating to the fundamental topics in real analysis.
* In this course mathematical exposition will be emphasized and solutions to most of the problems will be *proofs*. Students will be expected to understand and produce proofs. We will expect students to write proofs in complete, well-organized, and grammatically correct sentences (albeit using symbols).
* The Mathematics Subgroup has agreed on the following learning goals for this course:
  + Students will be able to prove convergence and divergence of limits using the ε−δ definition.
  + Students will be able to prove basic theorems about the notions of completeness, compactness and connectedness.
  + Students will be able to prove basic facts about derivatives and their properties.
  + Students will be able to prove basic facts about infinite series of functions.
  + Students will be able to write the definition of the Riemann integral and use it to compute a Riemann Integral of a function in elementary cases.
  + Students will demonstrate a rigorous understanding and working knowledge of the main concepts, theorems and techniques of real analysis by using the definitions, theorems, and examples to prove or disprove given statements.
* Additional Instructor References
  + S. Abbott, Understanding Analysis
  + G. Folland, Real Analysis
  + W. Rudin, Real and Complex Analysis
  + H.L. Royden, Real Analysis (2nd Ed.)
  + C. Apostol, Mathematical Analysis (2nd Ed.)
  + James R. Kirkwood, An Introduction to Analysis, Second Edition
  + R. G. Bartle and D. R. Sherbert, Introduction to Real Analysis

**Approximate Pace of Coverage:** 18 required sections in 36 class meetings (after accounting for test days, etc.) 🡪 approximately 2 sections per week on average, though some topics will take more and some less time, as reflected in the topics section above.

**Method of Instruction:** Methods may include lecture, group work, discussion of examples, and must include a significant opportunity for students to improve on their writing of proofs.

**Evaluation Procedures:** Possible methods include examinations, quizzes, homework problems, and a final examination.

# **General Education: Writing Intensive:** *The following language should appear in the syllabus for this course.*

# This is a General Education course that satisfies the Writing Intensive requirement. Mathematics 452 contains requirements and learning activities that promote students' abilities to...

1. practice the processes and procedures for creating and completing successful writing in their fields;
2. understand the main features and uses of writing in their fields;
3. adapt their writing to the general expectations of readers in their fields;
4. make use of the technologies commonly used for research and writing in their fields; and
5. learn the conventions of evidence, format, usage, and documentation in their fields.

Topics below which include such requirements and learning activities are indicated below using lowercase, boldface letters **a.-e.** corresponding to these requirements.

**Course Outline of the Major Topics and Subtopics:**

* The real number system and an introduction to proof. **a., b., c., d., e.**
* Elementary Topology—open/closed sets, countability, boundedness, compactness. **a., b., c., d., e.**
* Functions, Sequences, and Limits. **a., b., c., d., e.**
* Continuity. **a., b., c., d., e.**
* Differentiation. **a., b., c., d., e.**
* Integration. **a., b., c., d., e.**
* Vectors and Curves. **a., b., c., d., e.**
* Infinite Series. **a., b., c., d., e.**

**Additional Information about Writing Assignments:** In accordance with criteria **a.**, **b.**, **c.**, **d.**, and **e.**, this course provides the rigorous underpinnings of proof construction and writing that are expected of students planning to attend graduate school in mathematics. The abstracts and proofs that students write in this course constitute the vast majority of their grade. One such abstract/proof pair is given below as an example of the type of writing required in this course:

**Abstract:** In the following proof, we show that if a function , from set *S* to *T*, is a bijection, then its inverse must also be a bijection. To accomplish this, we begin by assuming that  is a bijection and then show that this assumption leads necessarily to the conclusion that  is a bijection. Hence, we begin with the knowledge that  has the following four properties:

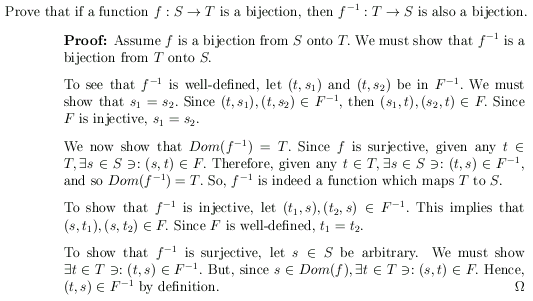
1. It is well-defined,
2. Its domain is all of the set *S*,
3. It is injective, and
4. It is surjective.

We must then prove that its inverse has the following four qualities:

1. It is well-defined,
2. Its domain is all of the set *T*,
3. It is injective, and
4. It is surjective.

We note that since  is injective, this will lead us to the conclusion that its inverse is well-defined, and the fact that  is well-defined will lead us to the conclusion that  is injective. Likewise, the fact that  is surjective leads to the conclusion that its inverse has *T* as its domain, and the fact that the domain of  is the set *S* leads to the conclusion that  is surjective.

**Proof:**

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**Minnesota Transfer Curriculum:** Not Applicable

**MnSCU Learning Outcomes:**

* Students will demonstrate proficiency in the basic methods of proof, including induction, as they apply to problems in analysis.
* Students will demonstrate a rigorous knowledge of the main results involving cardinality, completeness, sequences, continuity, and differentiability.
* Students will demonstrate an ability to explore problems from elementary analysis, make clear and precise conjectures, and rigorously prove their results.
* Students will be able to apply the topological structure of the real line in constructing mathematically rigorous proofs.
* Students will be able to prove the Mean Value Theorem and the Taylor’s Theorem and apply them in approximating functions.
* Students will be able to use the definition of the Riemann integral, prove elementary properties of the integral and the Fundamental Theorem of Calculus.
* Students will demonstrate a rigorous knowledge of main results involving cardinality, completeness, sequences, continuity, integrability, and differentiability.

**Last Revised:** Spring 2013 by the Mathematics Subgroup