WINONA STATE UNIVERSITY

COLLEGE OF SCIENCE AND ENGINEERING

DEPARTMENT OF MATHEMATICS AND STATISTICS

**Course Outline – MATH 453**

**Course Title:** Advanced Calculus II

**Number of Credits:** 3

**Catalog Description:** A systematic approach to the theory of differential and integral calculus for functions and transformations in several variables. Prerequisites: MATH 452 – Advanced Calculus I. Offered spring semesters of even numbered years (e.g. Spring 2012, Spring 2014, etc.)

**Possible Textbooks:** Real analysis for graduate students: measure and integration theory by Bass (free download)

**Topics Covered:**

1. Sequences of Functions
   1. Graphing
   2. Convergence and Divergence
   3. Pointwise Convergence
   4. Uniform Convergence
   5. Uniform Convergence and Integration
   6. Uniform Convergence and Differentiability
   7. Cauchy Criterion
   8. Sup Norm, L1 norm, L2 norm and norm
   9. Completeness of C[a, b]
2. Series and Power Series of Functions
   1. Weierstrass M-test
   2. Uniform convergence of series of functions
   3. Integrability and Differentiabilty
   4. Series approximation
   5. Applications
3. Metric Spaces
   1. Several examples of metrics
   2. Idea of Convergence in Metric Spaces
   3. Contraction Mapping Theorem
   4. Contraction Mapping Principle
   5. Applications
4. Normed Linear Spaces
   1. Banach Space
   2. Introduction to Hilbert Space
   3. Examples
   4. Natural marriage of Linear Algebra and Calculus
   5. Applications
5. Measure Theory
   1. Measures and Measurable Sets
   2. Properties
   3. C\*-Algebra
   4. Measurable Space
   5. Field of Sets
   6. Borel Set and C\*-Algebra
   7. Measurable Functions, Properties
   8. Sup and Inf of measurable functions
   9. A.E. Convergence
   10. Measurable Functions and Lebesgue Integral
   11. Cantor-Lebesgue function

**Listing of Sections in Departmental Text to be Covered (Name and Author of Text Here):**

**Remarks:** Students will give at least two “brief” presentations (approximately 10-12 minutes in length) and one major project presentation (approximately 20-25 minutes in length) as part of this course. These presentations are intended both to help them learn the material, and to give them practice and guidance in presenting mathematics orally. The presentations will be worth 50 points each, with points distributed as detailed on the Speaker Evaluation Form. The written major project will be worth an additional 100 points.

Students will need to use their discretion, and the instructor’s guidance, in deciding how much material to cover, which points to emphasize, and which points to omit from their presentation. The earlier the student plans and practices their presentation, the more help the instructor can offer them.

**Approximate Pace of Coverage:**

**Method of Instruction:** Lecture-presentation, discussion, question-answer sessions, group work.

**Evaluation Procedure:** Homework, brief oral presentations, quizzes, midterm exams, major project and presentation, and a final exam.

# **General Education: Mathematics/Statistics Intensive:** *The following language should appear in the syllabus for this course.*

# This is a General Education course that satisfies the Oral Intensive requirement. Mathematics 453 contains requirements and learning activities that promote students' abilities to...

1. **Earn significant course credit through extemporaneous oral presentations;**

Typically, students will be required to give at least three oral presentations to the class: one “major” presentation of an expository research paper, and at least two “minor” presentations either summarizing a short (5-10 page) section of the text, or explaining the solution to a series of related mathematical problems. These presentations will be worth roughly 15% to 25% of the student’s final grade, although this will vary somewhat depending upon the instructor and year. Typically, a full week of the semester is used for oral presentations of the “major” student projects.

1. **Understand the features and types of speaking in their disciplines;**

Students in this course are required to learn and perform four different types of speaking in this course: (1) the discussion necessary for a group of 2 to 4 students to solve mathematical problems; (2) presenting homework solutions to the class; (3) presenting brief summaries of section topics taken from the text or independent research; and (4) presenting the results of a major expository research project to the class.

The four types of speaking about mathematics listed above reflect four primary types of speaking needed by mathematicians and mathematics teachers. The first type, discussion among peers about mathematical concepts, examples, and arguments, is the primary means of progress in mathematical research, and is also a highly effective means of helping students at any level construct their own mathematical knowledge. Even to mathematics majors, mathematics is essentially a foreign language. This course is intended to help students learn to use and communicate mathematical terminology and arguments correctly and at a level of rigor and following the stylistic standards appropriate to the discipline.

The second type of speaking, presenting a solution to an audience, corresponds to the type of presentation that a teacher would give to a class after they have had time to work on a problem, or a presentation that a mathematician might give at a specialized conference. The audience is assumed to already know a significant amount of background and terminology, but a complete, step-by-step explanation of the specific claim must be given. This type of speaking requires the highest level of rigor out of the four types. This type of speaking is especially useful for students planning to go on to graduate school, who constitute a significant portion of the students who are enrolled in Advanced Calculus II.

The third and fourth types of speaking, presenting the results of a minor or major research project, correspond to the type of talk one would give for a more general audience that is presumed to have some general background knowledge but no specialized knowledge of the topic. In this type of speaking, a large amount of information is condensed and summarized to an introduction to the main themes of a topic.

1. **Adapt their speaking to field-specific audiences;**

Students in Advanced Calculus II learn to adapt their speaking to communicate effectively with (1) “collaborators,” i.e., students working together on the same problems, (2) “experts,” i.e., students in the same class working on different problems who have read or skimmed the background info but still need to hear details of the speaker’s specific work or topic, and (3) “non-experts” with only minimal background in the topic, as described in item **b.** above.

1. **Receive appropriate feedback from teachers and peers, including suggestions for improvement;**

Students receive immediate feedback from peers during discussion done in groups. The instructor circulates throughout the class during this group work, confirming when terminology and arguments are appropriate, and offering corrections, hints, and suggestions for improvement.

The instructor also comments orally after problem solutions and section summaries are presented to the class, and students receive written grades with feedback, evaluation, and suggestions for improvements on various aspects of their presentations.

For the major expository project and presentation, the instructor offers comments on a preliminary outline, then comments on a first draft, with suggestions for improvement and suggestions on the oral presentation, and comments on the final presentation.

1. **Make use of the technologies used for research and speaking in the fields; and**

Students typically use the blackboard or whiteboard to present solutions to homework problems; some use overhead slides or printed handouts to supplement their explanations. Some will learn to use the TeX or LaTeX mathematical software to typeset their solutions. They often use a graphing calculator or a computer algebra system (such as Mathematica) to help them solve the problems and present solutions. For their major project presentations, many students use Power Point (sometimes with audio supplements), some students use overhead slides, some use printed handouts, and some use the blackboard-- in approximately the same proportions as would be represented at a professional conference for mathematicians or mathematics teachers.

1. **Learn the conventions of evidence, format, usage, and documentation in their fields.**

Students must learn the current conventions and internationally held standards of evidence, format, usage, and documentation in the field of mathematics.

In reading about different mathematical topics and solving mathematical problems, students must learn how to move through the process of communicating their informal intuitions based on concrete examples, to developing formal, rigorous proofs, and then finally to explaining their proofs in ways that others can understand. That is, the type of speaking that is most effective for “discovering” a theorem and/or its proof is very different from the type of speaking that is required for presenting a formal statement of a theorem and its proof. Students must learn how to carry out both types of speaking, and learn to recognize which type of speaking (namely, what level of formality in terms of evidence and usage) is appropriate in different situations.

**Minnesota Transfer Curriculum:** Not Applicable

**MnSCU Learning Outcomes:**

* Students will be able to perform numeric and symbolic computations with sequence of functions.
* Students will be able to construct and apply symbolic and graphical representations of series of functions
* Students will be able to make clear distinction between pointwise and uniform convergence with analytical proofs.
* Students will be able to use technology appropriately to analyze the problems related to Calculus, Linear Algebra and Abstract Algebra in this course.
* Students will be able to state and apply mathematical definitions and theorems.
* Students will experience reading and writing proofs.
* Students will be able to prove fundamental theorems.
* Students will gain a rigorous understanding and working knowledge of the main concepts, theorems and techniques.
* Students will be able to construct and present a rigorous mathematical argument.
* Students will develop a clear mathematical writing style.
* Students will learn to be responsible for explaining solutions and communicating in writing and speaking.
* Students will give oral presentations on homework solutions, and on special topics not covered in the usual lectures, with feedback from teachers and peers and suggestions for improvement.

**Last Revised:** Spring 2013 by the Mathematics Subgroup