Einstein tells us that the whole of science is nothing more than a refinement of everyday thinking. It is easy in mathematics to bypass the “thinking” part, in favor of calculating, but to do so misses the whole point. One way to make sure you’re thinking (as opposed to just calculating) is to communicate what you’re thinking. Secondly, while most math graduates will never in their lives after graduation have to compute the cosets of a normal subgroup, almost all will be required to communicate technical information to people who don’t have the background to understand it. This is very hard to learn, but it begins with speaking and writing clearly among ourselves. For these reasons, good mathematical writing will be required and included in the grade for every assignment in this course. The remainder of this document will describe what I mean by “good mathematical writing.”

1 The Central Thing

• What you write must make sense. Try reading aloud what you have written. Think about it. Have you said anything ridiculous (it’s easier to do than it sounds, and I routinely root out ridiculous statements from my writing)? If you replaced every technical term with “This one thing” or “this other thing,” the people at the campus writing center should be able understand your sentences. All the rest is elaboration of this one point, that what you write must make sense, and without it, nothing else matters.

2 Language Issues

• At all times, write in complete sentences. Luckily, an equation is a sentence. Firmly fix this in your mind. Of course, it can also be used as a clause in a larger sentence: “If $2x^2 + 3 = 11$, then $x = \pm 2.$” Notice the correct connecting words (“if” and “then”) in the previous example. You should be able to read your work aloud verbatim and have it sound connected and flow smoothly.

• Spelling and grammar should be standard, except where good reason for a deviation exists.

• The advice of Strunk and White (Elements of Style, 4ed) is to be followed religiously:

  Vigorous writing is concise. A sentence should contain no unnecessary words, a paragraph no unnecessary sentences, for the same reason that a drawing should have no unnecessary lines and a machine no unnecessary parts. This requires not that the writer make all sentences short, or avoid all detail and treat subjects only in outline, but that every word tell.

By like token, the same authors wisely counsel, “Avoid the elaborate, the pretentious, the coy, and the cute. Do not be tempted by a twenty-dollar word when there is a ten-center handy, ready, and able.” While over-used, under-specific words are generally less desirable, one distinguishing mark I have seen in the best-educated people I know is that they can, when they wish, talk more plainly than anyone else.

• In professional writing (as in this class), the third person is usually preferred over the second, and slightly over the first, but avoiding artificial language is more important. Also, the active voice is usually preferred over the passive, but again, good expression is paramount. For instance, I could not figure out an equally honest and expressive way to write the last two sentences in the active voice, and to write the present sentence without the first person would be dishonest.
• Beware the technical terms “obvious,” “clear,” “trivial,” and their ilk. A friend of mine once remarked that the mathematical meaning of “obviously” was, “A human could prove it in a finite amount of time without any specialized knowledge.” You are almost always better off to either explain the conclusion or cite where you take it from (Theorem X.y in the book; Learned in MATH 165; etc.).

3 Audience Issues

• You will, except when otherwise instructed, be writing with the class as your audience, not just the instructor. You may assume that your audience understands the material from prerequisite courses and the present course to date (although it’s not a bad idea to remind the reader of more obscure points), but you should not assume that the reader is familiar with material later in the course, material from the book that we haven’t read, out-of-class conversations, or any other source not common to all students in the class.

4 Mathematical Issues

• A common abuse of language uses “=” to transition from one line to another in a calculation. That symbol should only be used to mean that two things are identical (e.g. 3 + 2 = 2 + 3).

• A proof should clearly describe what it is proving. In an involved argument, it sometimes helps to give an overview of the strategy before starting the line-by-line proof. All assumptions should be stated. The proof should have a clear end, at which the reader can tell how the foregoing argument establishes what was to be proved.

• Names should call to mind the thing being named, or at least should not contradict good sense. Re-using the same name within the same or a related argument is to be avoided.

• Do not let anything in this guide dissuade you from the frequently invaluable expedients of charts, tables, equations, formulae, pictures, or such. Use them when they’re useful. Just make sure that you explain them, too.