Problem 4.12 Prove that \((1 - 3x) = \sqrt{5x - 1}\) implies \(x = 2/9\)

Proof. Suppose \(x\) satisfies \((1 - 3x) = \sqrt{5x - 1}\). Squaring both sides yields

\[
\begin{align*}
5x - 1 &= (1 - 3x)^2 \\
5x - 1 &= 1 - 6x + 9x^2 \\
0 &= 2 - 11x + 9x^2.
\end{align*}
\]

Using the quadratic formula gives

\[
x = \frac{11 \pm \sqrt{11^2 - 4 \cdot 2 \cdot 9}}{2 \cdot 9} = \frac{11 \pm \sqrt{121 - 72}}{18} = \frac{11 \pm 7}{18} = 1, \frac{2}{9}.
\]

However, the case of \(x = 1\) must be thrown out since

\[
(1 - 3 \cdot 1) = -2 \neq 2 = \sqrt{5 \cdot 1 - 1}.
\]

Hence \(x = \frac{2}{9}\). \qed