Section X.4 Homework

ORAL:

1. (a) Solve $3x^5 + 4x^4 - 2x^3 + x - 7 \equiv 0 \mod 5$ by testing a finite number of guesses.
   (b) Lift your solution(s) to the above to solve $3x^5 + 4x^4 - 2x^3 + x - 7 \equiv 0 \mod 25$.
   (c) Lift your solution(s) to the above to solve $3x^5 + 4x^4 - 2x^3 + x - 7 \equiv 0 \mod 125$.
   (d) Solve $3x^5 + 4x^4 - 2x^3 + x - 7 \equiv 0 \mod 13$ by testing a finite number of guesses.
   (e) Use Chinese Remainder Theorem to solve $3x^5 + 4x^4 - 2x^3 + x - 7 \equiv 0 \mod 1625$.

2. (a) A solution to $x^6 + 3x + 110 \equiv 0 \mod 121$ is $x \equiv 44 \mod 121$. Use Hensel's Lemma to determine if this can be lifted to many, lifted to one, or not lifted to solutions mod 1331.
   (b) A solution to $x^6 + 18x^3 + 26 \equiv 0 \mod 9$ is $x \equiv 4 \mod 9$. Use Hensel's Lemma to determine if this can be lifted to many, lifted to one, or not lifted to solutions mod 27.
   (c) A solution to $x^6 + 18x^3 + 26 \equiv 0 \mod 9$ is $x \equiv 5 \mod 9$. Use Hensel's Lemma to determine if this can be lifted to many, lifted to one, or not lifted to solutions mod 27.

WRITTEN:

3. Solve
$$x^3 - 216 \equiv 0 \mod 5929$$

4. Prove that if a number n is given in base b by $n = a_k a_{k-1} \ldots a_3 a_2 a_1 a_0$ then n is divisible by $b-1$ if and only if $a_k + a_{k-1} + \cdots + a_2 + a_1 + a_0$ is divisible by $b-1$. (Note: This generalizes the idea that a base-10 number is divisible by 9 iff the sum of its digits are.)