## Math 280 Problems for September 21

## Pythagoras Level

Problem 1: The set $S$ contains ten numbers. The mean of the numbers in $S$ is 23 . The mean of the six smallest numbers in $S$ is 15 . The mean of the six largest numbers in $S$ is 30 . What is the median of the numbers in $S$ ?

Problem 2: In the figure below, $A$ and $B$ are the points $(2,0)$ and $(2,5)$ respectively ( $O$ is the origin). If right triangle $O A B$ is flipped about its hypotenuse as shown, what is the slope of the line through $O$ and $A^{\prime}$ ?


## Newton Level

Problem 3: Let $f_{1}(x)=f(x)=\frac{1}{1+2 x}$. Then for $n>1$, left $f_{n}(x)=f\left(f_{n-1}(x)\right)$. So, for example, $f_{3}(x)=f(f(f(x)))$. Compute $f_{7}^{\prime}(-1)$.

Problem 4: Find the limit

$$
\lim _{n \rightarrow \infty}\left[\frac{\left(1+\frac{1}{n}\right)^{n}}{e}\right]^{n}
$$

## Wiles Level

Problem 5: If $A$ is the matrix $\left(\begin{array}{cc}1 & 3 \\ -1 & 1\end{array}\right)$, determine the series:

$$
A-\frac{1}{3} A^{2}+\frac{1}{9} A^{3}+\cdots+\left(-\frac{1}{3}\right)^{n} A^{n+1}+\cdots
$$

Problem 6: Compute the area of the region which lies between the $x$-axis and the curve, $y=e^{-x} \sin (\pi x)$, for $x \geq 0$.

