Math 280 Problems for September 21

Pythagoras Level

Problem 1: The set \( S \) contains ten numbers. The mean of the numbers in \( S \) is 23. The mean of the six smallest numbers in \( S \) is 15. The mean of the six largest numbers in \( S \) is 30. What is the median of the numbers in \( S \)?

Problem 2: In the figure below, \( A \) and \( B \) are the points (2, 0) and (2, 5) respectively (\( O \) is the origin). If right triangle \( OAB \) is flipped about its hypotenuse as shown, what is the slope of the line through \( O \) and \( A' \)?

Newton Level

Problem 3: Let \( f_1(x) = f(x) = \frac{1}{1+2x} \). Then for \( n > 1 \), left \( f_n(x) = f(f_{n-1}(x)) \). So, for example, \( f_3(x) = f(f(f(x))) \). Compute \( f'_7(-1) \).

Problem 4: Find the limit

\[
\lim_{n \to \infty} \left[ \frac{(1 + \frac{1}{\pi})^n}{e} \right]^n.
\]

Wiles Level

Problem 5: If \( A \) is the matrix \( \begin{pmatrix} 1 & 3 \\ -1 & 1 \end{pmatrix} \), determine the series:

\[
A - \frac{1}{3}A^2 + \frac{1}{9}A^3 + \cdots + \left( -\frac{1}{3} \right)^n A^{n+1} + \cdots
\]

Problem 6: Compute the area of the region which lies between the \( x \)-axis and the curve, \( y = e^{-x} \sin(\pi x) \), for \( x \geq 0 \).