# Math 280 Problems for September 28

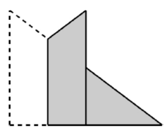
## Pythagoras Level

**Problem 1:** Suppose 100 people are each assigned a different seat on an airplane with 100 seats. The passengers are seated one at a time. The first person loses his boarding pass and sits in one of the 100 seats chosen at random. Each subsequent person sits in their assigned seat if it is unoccupied, and otherwise chooses a seat at random from among the remaining empty seats. Determine, with proof, the probability that the last person to board the plane is able to sit in her assigned seat.

**Problem 2:** A jar contains 600 jelly beans, 100 red, 200 green, and 300 blue. These are drawn randomly from the jar, one at a time, without replacement. What is the probability that the first color to be exhausted is red?

## Newton Level

**Problem 3:** Consider a triangular sheet of paper with vertices at the points $(0,0)$, $(4,0)$ and $(0,3)$. By making a single vertical crease in the paper and then folding the left portion of the triangle over the crease so that it overlaps the right portion we obtain a polygon with five sides, colored grey. Find the smallest possible area of this resulting polygon.



**Problem 4:** A smooth curve crosses the $y$-axis at the point $(0,2)$ and has the following curious property. Given any point $P$ on the curve, the tangent line to the curve at $P$ crosses the $x$-axis at a point $Q$ exactly 2011 units to the right of $P$. (In other words, the $x$-coordinate of $Q$ is 2011 more than the $x$-coordinate of $P$.) Determine the area of the region in the first quadrant bounded by the $x$-axis, the $y$-axis, and this curve, explaining how you found your answer.

## Wiles Level

**Problem 5:** Observe that $(1)(4)(7) = 28$ is one greater than a multiple of 9, while $(2)(5)(8) = 80$ is one less than a multiple of 9. Confirm that this phenomenon persists; in other words, prove that for all $n \geq 1$ we have

$$(1)(4)(7)(10)\cdots(3\cdot 3^{n-1}-2) \equiv 1 \mod 3^n,$$

$$(2)(5)(8)(11)\cdots(3\cdot 3^{n-1}-1) \equiv -1 \mod 3^n.$$

**Problem 6:** Prove that

$$e^{e^x} \geq e^{2x+1} - xe^{x+1}$$

for all real $x$.