Practice Problems on Volumes of Solids of Revolution

Find the volume of each of the following solids of revolution obtained by rotating the indicated regions.

a. Bounded by \( y = 1/x, \ y = 2/x, \) and the lines \( x = 1 \) and \( x = 3 \) rotated about the \( x \)-axis.

\[
\text{Disk: } V = \pi \int_1^3 \left( \frac{2}{x} \right)^2 - \left( \frac{1}{x} \right)^2 \, dx = 2\pi
\]

b. The region in the preceding problem rotated about the line \( y = -1. \)

\[
\text{Disk: } V = \pi \int_1^3 \left( \frac{2}{x} + 1 \right)^2 - \left( \frac{1}{x} + 1 \right)^2 \, dx = \pi (2 + 2 \ln 3)
\]

c. Bounded by \( y = 1/x, \ y = 2/x, \) and the lines \( x = 1 \) and \( x = 3 \) rotated about the \( y \)-axis. (Previous region)

\[
\text{Disk: } V = \pi \int_1^2 \left( \frac{(2/y)^2 - 1^2}{2} \right) \, dy + \pi \int_1^{1/2} \left( (2/y)^2 - (1/y)^2 \right) \, dy + \pi \int_{1/3}^{2/3} \left( 3^2 - (1/y)^2 \right) \, dy = 4\pi
\]

\[
\text{Cylindrical Shell: } V = 2\pi \int_1^3 x \left( \frac{2}{x} - 1/x \right) \, dx = 4\pi
\]

d. Use the Cylindrical Shell Method to find the volume of the solid obtained by rotating the region bounded by \( y = \sqrt{1-x^2}, \) the \( x \)-axis, and the \( y \)-axis in the first quadrant rotated about the \( y \)-axis.

\[
V = 2\pi \int_0^1 x \sqrt{1-x^2} \, dx = -\frac{2}{3} \pi (1 - x^2)^{3/2} \bigg|_0^1 = 2\pi/3
\]

e. Bounded by \( y = e^{2x} \) and \( y = e^{-2x} \) on \([0, 2]\) rotated about the \( x \)-axis.
Disk: \( V = \pi \int_0^2 \left( (e^{2x})^2 - (e^{-2x})^2 \right) \, dx = \frac{e^8 + e^{-8} - 2}{4} \pi \)

f. The region in the preceding problem rotated about the y-axis. (Set up the integral only, but do not integrate.)

Cylindrical Shell: \( V = 2\pi \int_0^2 x(e^{2x} - e^{-2x}) \, dx \)

Disk: \( V = \pi \int_1^4 \{ 2^2 - \left( \frac{1}{2} \ln y \right)^2 \} \, dy + \pi \int_{e^{-4}}^1 \{ 2^2 - \left( -\frac{1}{2} \ln y \right)^2 \} \, dy \)

g. The region above \( y = \sin x \) and below \( y = 1 \) on \([0, \pi/2]\) (yellow region below) rotated about the x-axis. (Set up the integral only, but do not integrate.)

Disk: \( V = \pi \int_0^{\pi/2} [1 - \sin^2 x] \, dx \)

Cylindrical Shell: \( V = 2\pi \int_0^1 x \arcsin x \, dx \)

h. The region below \( y = \sin x \) and above the x-axis on \([0, \pi/2]\) (green region in the preceding problem) rotated about the y-axis. (Set up the integral only, but do not integrate.)

Cylindrical Shell: \( V = 2\pi \int_0^{\pi/2} x(\sin x) \, dx \)

Disk: \( V = \pi \int_0^{\pi/2} \{(\pi/2)^2 - (\arcsin y)^2 \} \, dy \)

i. The region below \( y = \sin x \) and \( y = \cos x \) but above the x-axis on \([0, \pi/2]\). (Set up the integral only, but do not integrate.)
j. The region in the preceding problem rotated about the y-axis. (Set up the integral only, but do not integrate.)

\[
\text{Disk: } \pi \int_0^{\sqrt{2}/2} y \{\arccos y - \arcsin y\} \, dy + \pi \int_{\sqrt{2}/2}^{\pi/4} \{y \arccos y - (\arcsin y)^2\} \, dy
\]

\[
\text{Cylindrical Shell: } 2\pi \int_0^{\sqrt{2}/2} x \sin x \, dx + 2\pi \int_{\sqrt{2}/2}^{\pi/4} x \cos x \, dx
\]

k. The region bounded by \( y = 4 - x^2 \) and \( y = x^4 + 2 \) rotated about the x-axis.

\[
\text{Disk: } A = \int_{-1}^1 \{(4 - x^2)^2 - (x^4 + 2)^2\} \, dx = \frac{776\pi}{45}
\]