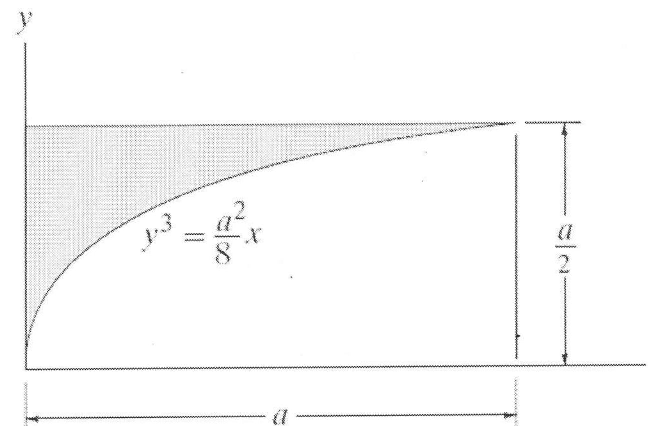
CME 250 – Statics
Exam #8 (12/05/2023)

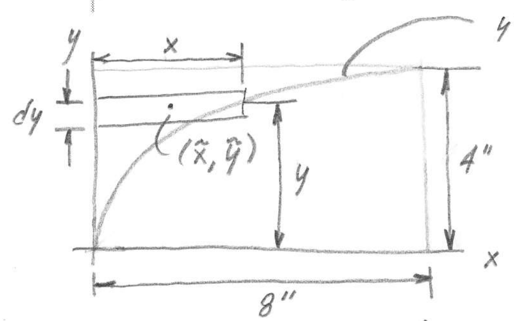
StarID or TechID (no names) Grading

Show your work (you will not receive any credit if all you have is a final answer, right or wrong).
Do one of the two problems shown below (the second problem is on the back).

1. Determine the location of the centroid $(\overline{x}, \overline{y})$ of the area. Set a = 8 inches.



$dA = (4-y)\,dx = (4-2x^{1/3})\,dx$

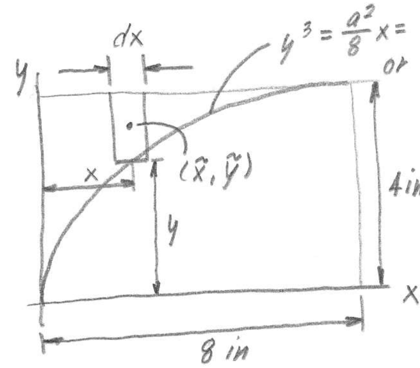


$y^3 = \frac{a^2}{8}x = 8x$

or $x = \frac{y^3}{8}$

$dA = x\,dy = \frac{y^3}{8}\,dy$

OR



$y^3 = \frac{a^2}{8}x = 8x$

or $y = 8^{1/3}x^{1/3} = 2x^{1/3}$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^4 \frac{x}{2}\frac{y^3}{8}\,dy}{\int_0^4 \frac{y^3}{8}\,dy} = \frac{\int_0^4 \frac{y^3}{16}\frac{y^3}{8}\,dy}{\int_0^4 \frac{y^3}{8}\,dy}$$

$$= \frac{\frac{1}{128}\int_0^4 y^6\,dy}{\frac{1}{8}\int_0^4 y^3\,dy} = \frac{\overset{(3)}{18.286\text{ in}^3}}{\underset{(3)}{8\text{ in}^2}} = \boxed{2.29\text{ in}}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^4 y\,dA}{\text{same as above}} = \frac{\int_0^4 y\frac{y^3}{8}\,dy}{8\text{ in}^2}$$

$$= \frac{\frac{1}{8}\int_0^4 y^4\,dy}{8\text{ in}^2} = \frac{\overset{(3)}{25.6\text{ in}^3}}{\underset{(1)}{8\text{ in}^2}} = \boxed{3.20\text{ in}}$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^8 x(4-2x^{1/3})\,dx}{\int_0^8 (4-2x^{1/3})\,dx}$$

$$= \frac{18.286\text{ in}^3}{8\text{ in}^2} = \boxed{2.29\text{ in}}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^8 \left(y + \frac{4-y}{2}\right)(4-2x^{1/3})\,dx}{\text{same as above}}$$

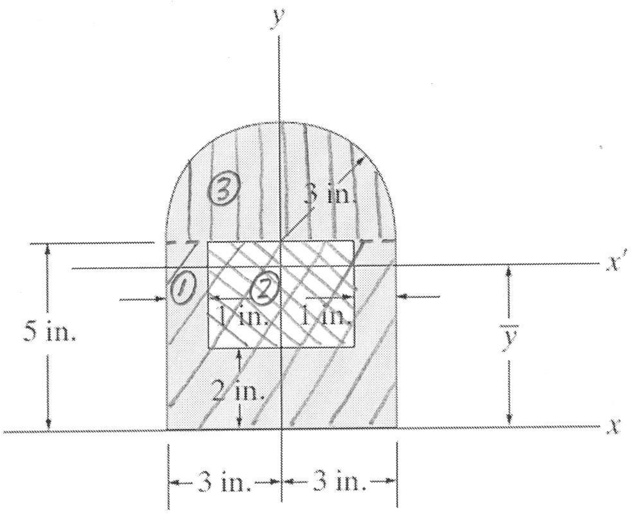
$$= \frac{\int_0^8 \left(y+2-\frac{y}{2}\right)(4-2x^{1/3})\,dx}{8\text{ in}^2}$$

$$= \frac{\int_0^8 \left(\frac{y}{2}+2\right)(4-2x^{1/3})\,dx}{8\text{ in}^2}$$

$$= \frac{\int_0^8 (x^{1/3}+2)(4-2x^{1/3})\,dx}{8\text{ in}^2} = \frac{25.6\text{ in}^3}{8\text{ in}^2}$$

$$= \boxed{3.20\text{ in}}$$

2. Determine the moment of inertia of the composite area about the y-axis (the area is symmetric about the y-axis).



the y-axis aligns with the y centroidal axes of the three simple geometries, areas 1, 2, and 3 $\rightarrow$ the parallel-axis theorem is not needed (3)

$$I_y = \frac{1}{12}(5\,in)(6\,in)^3 \quad (2)$$

$$-\frac{1}{12}(3\,in)(4\,in)^3 \quad (2)$$

$$+\frac{1}{8}\pi(3\,in)^4 \quad (2)$$

$$= 90\,in^4 - 16\,in^4 + 31.81\,in^4$$

$$= 105.8\,in^4 \rightarrow \boxed{106\,in^4} \quad (1)$$