Title: Abstract Algebra I

Number of Credits: 3

Catalog Description: Axiomatic development of groups, rings, and fields. Prerequisite: MATH 327 – Foundations of Mathematics. Recommended: MATH 347 – Number Theory. Offered every fall semester.

Possible Textbooks:

- Contemporary Abstract Algebra, 6th edition by Joseph Gallian
- A First Course in Abstract Algebra, 2nd edition by Joseph Rotman
- Abstract Algebra, 3rd Edition by I. N. Herstein
- Abstract Algebra-An Introduction, 2nd edition, by Thomas Hungerford
- Elements of Modern Algebra, 6th edition by Jimmie Gilbert and Linda Gilbert
- A First Course in Abstract Algebra, 6th edition by John B. Fraleigh

Topics Covered:

A. Introduction to Abstract Algebra and its Historical Origins (optional)
   1. Questions of constructibility in geometry
   2. Solving equations
   3. Comparing and contrasting properties of mathematical structures

B. Review of Foundations of Mathematics
   1. Induction, the well-ordering principle
   2. Other proof techniques
   3. Basic number theory
      a. Factoring and divisibility
      b. Modular arithmetic

C. An Introduction to Groups
   1. Finite groups
   2. Subgroups
   3. Cyclic groups
   4. Permutation groups

D. Group Homomorphisms
   1. Group isomorphisms
   2. Homomorphism theorems and properties
   3. Cosets and Lagrange's Theorem
   4. Factor groups
   5. Normal subgroups
E. An Introduction to Rings and Fields
   1. Finite and infinite rings
   2. Subrings and unity
   3. Integral domains and fields
   4. Associates, irreducibles, and factorization
   5. Ideals
F. Ring Homomorphisms
   1. Ring isomorphism
   2. Homomorphism theorems and properties
   3. The kernel
   4. Ring cosets
   5. Factor rings
G. Other topics may include (optional)
   1. Historical biographies of algebraists
   2. Coding theory
   3. Cryptography

Listing of Sections in Departmental Text to be Covered (Name and Author of Text Here): No Departmental Text required for this Course

Remarks: Students would always need a review of Discrete Math and Linear Algebra material.

Approximate Pace of Coverage: 2 weeks on review 4-5 weeks on Groups, 3-4 weeks on Rings about 2 weeks on Fields.

Method of Instruction: Lecture-presentation, discussion, question-answer sessions, use of calculators/computers, group work and/or paper presentations.

Evaluation Procedure: Homework, quizzes, projects, midterm exams, and a final exam and a research article.

General Education: Writing Intensive: The following language should appear in the syllabus for this course.

This is a General Education course that satisfies the Writing Intensive requirement. Mathematics 452 contains requirements and learning activities that promote students' abilities to...

a. practice the processes and procedures for creating and completing successful writing in their fields;

This course is a rigorous introduction to the concepts of Abstract Algebra. To successfully complete the course, the student is required to demonstrate not only an understanding of the mathematical concepts involved in Abstract Algebra, but also an ability to convey those concepts in concise written form, both formally
(proof) and informally (abstract). Mathematical proof represents a very precise writing style that has developed over two thousand years, and writing an abstract of a proof requires an understanding of, and an ability to articulate, the methods and strategies used to construct the proof. Proper use of this writing style requires a knowledge of the relevant terminology and a facility with the grammar and sentence structure that is germane to good expository writing. The student receives feedback on his or her written presentation of logical arguments throughout the semester, with the opportunity to refine both the proofs and abstracts.

b. **understand the main features and uses of writing in their fields;**
   It is in rigorous courses such as Abstract Algebra that the student’s conceptual understanding of mathematics is expanded into a rigorous understanding of the logical underpinnings of mathematical abstraction. This logical foundation, by its very nature, is inextricably interwoven with the precise writing that is used to express it. It is here that the student gains an awareness that proofs of mathematical theorems and propositions lie not in convincing pictures or clever examples, but in very precise and carefully applied logical analysis. Such analysis is only as clear as its exposition. A proof is not clear unless the reader has a prior organizational structure within which to interpret the proof. An abstract serves the purpose of providing the reader with this necessary tool.

c. **adapt their writing to the general expectations of readers in their fields;**
   Writing a mathematical proof is a very different type of writing compared to most other exposition. In this course, the successful student must learn to weave good sentence structure with mathematical formulae and symbolism in a way that brings clarity to the subject of the exposition. Particularly close attention must be paid to the implications of uni- and bi-conditional statements and the differences among theorems, conjectures, lemmas, and definitions. On the other hand, to write an abstract of a proof, the student must have a facility with these ideas that runs deeply enough to allow him/her to accurately present the essence of the thinking behind a proof without becoming excessively technical.

d. **make use of the technologies commonly used for research and writing in their fields;**
   When attempting to uncover patterns in analysis and abstraction, the student routinely makes use of various graphical and algebraic computer aids, such as graphing calculators or computer algebra systems. Additionally, there are special scripting languages for typesetting mathematical exposition, such as TeX and LaTeX. Either these or the equation editors in most popular word processors may be used to render proofs in this course. The use of typesetting tools is not a requirement of the course, but instead an option whose implementation is left to the discretion of the instructor.

e. **learn the conventions of evidence, format, usage, and documentation in their fields.**
As discussed above, the student encounters heavy use of mathematical terminology, mathematical reasoning, and the expository strategies unique to the field of mathematical analysis as well as the conventions for citing previously proven results, such as lemmas or theorems, in a mathematical proof.

**Minnesota Transfer Curriculum:** Not Applicable

**MnSCU Learning Outcomes:**
- Students will use definitions in abstract algebra and identify examples and non-examples.
- Students will write proofs of theorems and results in abstract algebra, using correct proof techniques and mathematical writing standards.
- Students will evaluate the correctness or incorrectness of an alleged "proof."
- Students will demonstrate the ability to move adeptly between specific examples or calculations and general abstract structures and results in abstract algebra.

**Last Revised:** Spring 2013 by the Mathematics Subgroup