University Studies Course Approval Proposal

Flag Requirements – Writing Flag

The Department of Mathematics and Statistics proposes the following course for inclusion in University Studies as a course satisfying the requirements for a Writing Flag. This was approved by the full department on Thursday, January 18, 2001.

Course: Advanced Calculus I (MATH 330), 4 s.h.

Catalog Description: A systematic approach to the theory of differential and integral calculus for functions and transformations in several variables. This is a University Studies course satisfying requirements for a Writing Flag. *Prerequisite:* MATH 210, and MATH 260.

This is an existing course, previously approved by A2C2.

Department Contact Person for this Course:

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General Discussion of University Studies – the Writing Flag in relation to MATH 330

University Studies: Writing Flag

Flagged courses will normally be in the student's major or minor program. Departments will need to demonstrate to the University Studies Subcommittee that the courses in question merit the flags. All flagged courses must require the relevant basic skills course(s) as prerequisites (e.g., the "College Reading and Writing" Basic Skill course is a prerequisite for Writing Flag courses), although departments and programs may require additional prerequisites for flagged courses. The University Studies Subcommittee recognizes that it cannot veto department designation of flagged courses.

The purpose of the Writing Flag requirement is to reinforce the outcomes specified for the basic skills area of writing. These courses are intended to provide contexts, opportunities, and feedback for students writing with discipline-specific texts, tools, and strategies. These courses should emphasize writing as essential to academic learning and intellectual development.

Courses can merit the Writing Flag by demonstrating that section enrollment will allow for clear guidance, criteria, and feedback for the writing assignments; that the course will require a significant amount of writing to be distributed throughout the semester; that writing will comprise a significant portion of the student's final course grade; and that students will have opportunities to incorporate readers' critiques of their writing.

These courses must include requirements and learning activities that promote students' abilities to:

a. **practice the processes and procedures for creating and completing successful writing in their fields;**
This course is a rigorous introduction to higher level mathematical analysis. To successfully complete the course, the student is required to demonstrate not only an understanding of the mathematical concepts involved in analysis, but also an ability to convey those concepts in concise written form. Mathematical proof represents a very precise writing style that has developed over several hundred years. Proper use of this writing style requires a knowledge of the relevant jargon and a facility with the grammar and sentence structure that is germane to good expositional writing. The student receives feedback on their written presentation of logical arguments throughout the semester.

b. **understand the main features and uses of writing in their fields;**
It is in rigorous courses such as analysis that the student's conceptual understanding of mathematics is expanded into a rigorous understanding of the logical underpinnings of mathematical abstraction. This logical foundation, by its very nature, is inextricably interwoven with the precise writing that is used to express it. It is here that the student gains an awareness that proofs of mathematical theorems and propositions lie not in convincing pictures or clever examples, but in very precise and carefully applied logical analysis. Such analysis is only as clear as its exposition.

c. **adapt their writing to the general expectations of readers in their fields;**
Writing mathematical proof is a very different type of writing from most other exposition. In this course, the successful student must learn to weave good sentence structure with mathematical formulae and symbolism in a way that brings clarity to the subject of the exposition. Particularly close attention must be paid to the implications of uni and bi-conditional statements and the differences between theorems, conjectures, lemmas, and definitions.

d. **make use of the technologies commonly used for research and writing in their fields; and**
When attempting to uncover patterns in analysis, the student routinely make use of various graphical and algebraic computer aids, such as graphing calculators or computer algebra systems. Additionally, there are special scripting languages for typesetting mathematical exposition, such as TeX and LaTeX. Either these or the equation editors in most popular word processors may be used to render proofs in this class. However, the use of typesetting tools is not a requirement of the course, but an option whose implementation is left to the discretion of the instructor.

e. **learn the conventions of evidence, format, usage, and documentation in their fields.**
As discussed above, the student encounters heavy use of mathematical jargon, mathematical reasoning, and the expositional strategies unique to the field of mathematical analysis as well as the conventions for citing previously proven results, such as lemmas or theorems, in a mathematical proof.

Winona State University
Department of Mathematics and Statistics
Course Outline—M330[1]

Course Title: Advanced Calculus I
Number of Credits: 4 s.h.
Prerequisite: Discrete Mathematics and Foundations (M210) & Multivariable Calculus (M260).
Grading: Grade only for all majors, minors, options, concentrations and licensures within the Department of Mathematics and Statistics. The P/NC option is available to others.
Course Description: A systematic approach to the theory of differential and integral calculus for functions and transformations in one and several variables.
Statement of Major Focus and Objectives of the Course: The major focus of this course is to introduce students to the logical underpinnings of mathematical analysis and to provide students with the ability to demonstrate a rigorous understanding of analysis by writing clear, accurate, and concise proofs.
Possible Texts:

- *Fundamental Ideas of Analysis* by Reed.
- *Advanced Calculus* by Buck.
- *Advanced Calculus* by Fulks.
- *Advanced Calculus* by Widder.
- *Advanced Calculus, a Friendly Approach* by Kosmala.
- *An Introduction to Analysis, 2nd edition* by Bartle and Sherbert.
- *Introduction to Real Analysis* by Gaughan.
- *Introduction to Real Analysis* by Schramm.
- *Principles of Mathematical Analysis* by Rudin.

Methods of Instruction: Lecture, Discussion, Problem Sets (possibly cooperative).
Course Requirements: None other than the text.
Evaluation Process: Tests, Quizzes, Problems Sets.
University Studies: *Writing Flag*

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[1] Prepared by Barry A. Peratt on February 9, 2001.

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These courses must include requirements and learning activities that promote students' abilities to:

a. practice the processes and procedures for creating and completing successful writing in their fields;
b. understand the main features and uses of writing in their fields;
c. adapt their writing to the general expectations of readers in their fields;
d. make use of the technologies commonly used for research and writing in their fields; and
e. learn the conventions of evidence, format, usage, and documentation in their fields.

Topics below which include such requirements and learning activities are indicated below using lowercase, boldface letters **a.-d.** corresponding to these requirements.

Course Outline of the Major Topics and Subtopics:

- The real number system and an introduction to proof. **a., b., c., d., e.**
- Elementary Topology—open/closed sets, countability, boundedness, compactness. **a., b., c., d., e.**
- Functions, Sequences, and Limits. **a., b., c., d., e.**
- Continuity. **a., b., c., d., e.**
- Differentiation. **a., b., c., d., e.**
- Integration. **a., b., c., d., e.**
- Vectors and Curves. **a., b., c., d., e.**
- Infinite Series. **a., b., c., d., e.**

Additional Information about Writing Assignments: In accordance with criteria **a.**, **b.**, **c.**, **d.**, and **e.**, this course provides the rigorous underpinnings of proof construction and writing that are expected of students planning to attend graduate school in mathematics. The proofs that students write in this course constitute the vast majority of their grade. Two such proofs are given below as an example of the type of writing required in this course:

4. Prove that a function $f : S \to T$ is a bijection iff f^{-1} is a bijection.

Proof: Assume f is a bijection from S onto T. We must show that f^{-1} is a bijection from T onto S. To see that f^{-1} is well-defined, let (t, s_1) and (t, s_2) be in F^{-1}. We must show that $s_1 = s_2$. Since $(t, s_1), (t, s_2) \in F^{-1}$, then $(s_1, t), (s_2, t) \in F$. Since F is injective, $s_1 = s_2$.

We now show that $Dom(f^{-1}) = T$. Since f is surjective, given any $t \in T, \exists s \in S \ni: (s, t) \in F$. Therefore, given any $t \in T, \exists s \in S \ni: (t, s) \in F^{-1}$, and so $Dom(f^{-1}) = T$. So, f^{-1} is indeed a function which maps T to S.

To show that f^{-1} is injective, let $(t_1, s), (t_2, s) \in F^{-1}$. This implies that $(s, t_1), (s, t_2) \in F$. Since F is well-defined, $t_1 = t_2$.

To show that f^{-1} is surjective, let $s \in S$ be arbitrary. We must show $\exists t \in T \ni: (t, s) \in F^{-1}$. But, since $s \in Dom(f), \exists t \in T \ni: (s, t) \in F$. Hence, $(t, s) \in F^{-1}$ by definition. Ω

6. Show that the cardinality of $\mathbb{Z}$ is the same as the cardinality of $\mathbb{Z}^+$ by finding the explicit formula for a bijective function $f : \mathbb{Z}^+ \to \mathbb{Z}$.

Proof: Define $f : \mathbb{Z}^+ \to \mathbb{Z}$ via

$$f(x) := \begin{cases} \frac{x}{2} & \text{if } x \text{ is even} \\ \frac{1-x}{2} & \text{if } x \text{ is odd} \end{cases}$$

We claim that f is one-to-one and onto.

To see that it is one-to-one, let $z \in \mathbb{Z}$ and suppose that $\exists x_1, x_2 \in \mathbb{Z}^+ \ni:$ $(x_1, z), (x_2, z) \in F$. We must show that $x_1 = x_2$. To see this first note $\frac{1-x}{2} \leq 0 \; \forall x \in \mathbb{Z}^+$ and $\frac{x}{2} > 0 \; \forall x \in \mathbb{Z}^+$. Thus, if $z \leq 0$, both x_1 and x_2 must be odd. In this case, $\frac{1-x_1}{2} = z = \frac{1-x_2}{2}$ and so $x_1 = x_2$. If $z > 0$, then both x_1 and x_2 must be even, in which case $\frac{x_1}{2} = z = \frac{x_2}{2}$, so that $x_1 = x_2$.

To see that f is onto, let $z \in \mathbb{Z}$. We must show that $\exists x \in \mathbb{Z}^+ \ni: \; f(x) = z$. If $z \leq 0$, let $x = 1 - 2z$. Since $-2z$ is even, $1 - 2z = -2z + 1$ is odd. Thus, $f(x) = \dfrac{1 - (-2z + 1)}{2} = z$. If $z > 0$, then let $x = 2z$. Since $2z$ is even, then $f(x) = 2z/2 = z$. Since z was arbitrary, this shows that f is onto. Ω

Approval/Disapproval Recommendations

Department Recommendation: Approved ____ Disapproved ____ Date ___

Chairperson Signature ________________________________ Date ______

Dean's Recommendation: Approved ____ Disapproved ____ Date ___

Dean's Signature ____________________________________ Date ______

*In the case of a Dean's recommendation to disapprove a proposal, a written rationale for the recommendation to disapprove shall be provided to USS.

USS Recommendation: Approved ____ Disapproved ____ Date ___

University Studies Director's Signature _____________________ Date ___

A2C2 Recommendation: Approved ____ Disapproved ____ Date ___

A2C2 Chairperson Signature ____________________________ Date ______

Faculty Senate Recommendation: Approved ____ Disapproved ____ Date ___

FA President's Signature ______________________________ Date ______

Academic VP's Recommendation: Approved ____ Disapproved ____ Date ___

VP's Signature _______________________________________ Date ______

President's Decision: Approved ____ Disapproved ____ Date ___

President's Signature _________________________________ Date ______