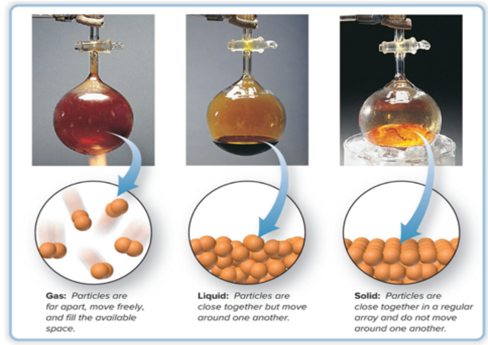
Gas Laws

Gases



© McGraw-Hill Education/Stephen Frisch, photographer

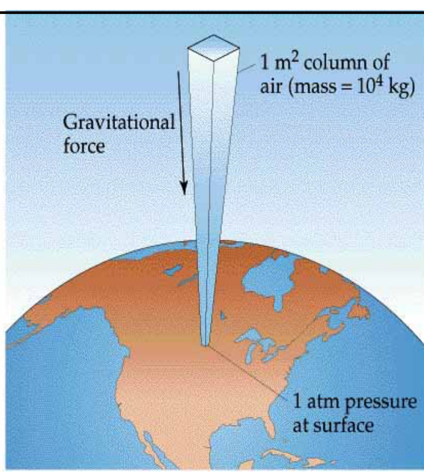
A Gas

- Uniformly fills any container.
- Mixes completely with any other gas
- Exerts pressure on its surroundings.

Pressure

- is equal to force/unit area
- SI units = Newton/meter^2 = 1 Pascal (Pa)
- 1 standard atmosphere = 101,325 Pa
- 1 standard atmosphere = 1 atm = 760 mm Hg = 760 torr

1atm = 760 mmHg



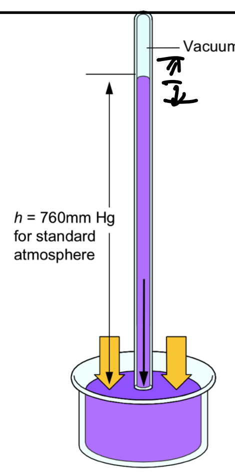


Table 5.1 Common Units of Pressure

Unit	Normal Atmospheric Pressure at Sea Level and 0°C
pascal (Pa); kilopascal (kPa)	1.01325×10^5 Pa; 101.325 kPa
atmosphere (atm)	1 atm*
millimeters of mercury (mmHg)	760 mmHg*
torr	760 torr*
pounds per square inch (lb/in^2 or psi)	14.7 lb/in^2
bar	1.01325 bar

*These are exact quantities; in calculations, we use as many significant figures as necessary.

5.8 Convert the following:

(a) 0.745 atm to mmHg $\quad 0.745 \text{ atm} \times \frac{760 \text{ mmHg}}{1 \text{ atm}} = 566 \text{ mmHg}$

(b) 992 torr to bar

(c) 365 kPa to atm

(d) 804 mmHg to kPa

$365 \text{ kPa} \times \frac{1 \text{ atm}}{101.325 \text{ kPa}} = 3.60 \text{ atm}$

Copyright © McGraw-Hill Education. All rights reserved. No reproduction or distribution without the prior written consent of McGraw-Hill Education.

Table 5.1 Common Units of Pressure

Unit	Normal Atmospheric Pressure at Sea Level and 0°C
pascal (Pa); kilopascal (kPa)	1.01325×10^5 Pa; 101.325 kPa
atmosphere (atm)	1 atm*
millimeters of mercury (mmHg)	760 mmHg*
torr	760 torr*
pounds per square inch (lb/in^2 or psi)	14.7 lb/in^2
bar	1.01325 bar

*These are exact quantities; in calculations, we use as many significant figures as necessary.

5.14 Convert each of the pressures described below to atm:

(a) At the peak of Mt. Everest, atmospheric pressure is only 2.75×10^2 mmHg.

(b) A cyclist fills her bike tires to 86 psi.

(c) The surface of Venus has an atmospheric pressure of 9.15×10^6 Pa.

(d) At 100 ft below sea level, a scuba diver experiences a pressure of 2.54×10^4 torr.

a.) $2.75 \times 10^2 \text{ mmHg} \times \frac{1 \text{ atm}}{760 \text{ mmHg}} = 0.36$

b.) $86 \text{ psi} \times \frac{1 \text{ atm}}{14.7 \text{ lb/in}^2} = 5.85 \text{ atm}$

Boyle's Law*

mole

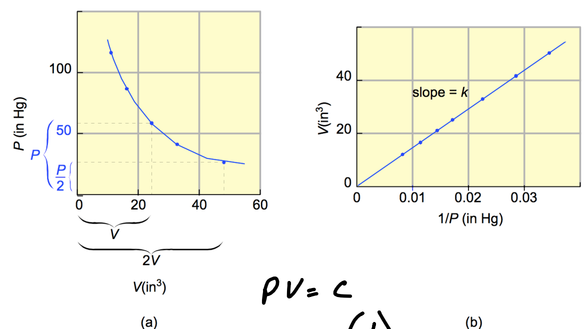
Pressure × Volume = Constant $\quad$ (T = constant)

$P_1V_1 = P_2V_2 \quad (T = \text{constant})$

$V \propto 1/P \quad (T = \text{constant})$

(*Holds *precisely* only at very low pressures.)

$P_1V_1 = C \qquad P_2V_2 = C$

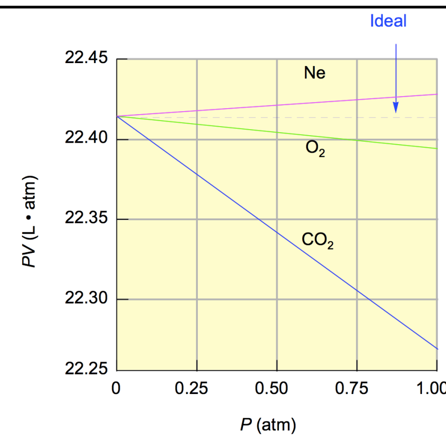


$PV = c$

$P = c\left(\frac{1}{V}\right)$

A gas that strictly obeys Boyle's Law is called an ideal gas.

low Pres and high Temp



Charles's Law

The volume of a gas is directly proportional to temperature, and extrapolates to zero at zero Kelvin.

$$V = bT \quad (P = \text{constant})$$

b = a proportionality constant

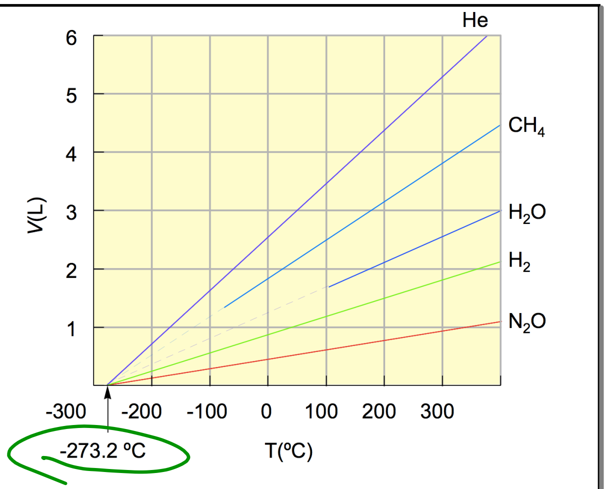
$V \quad T \quad P$

Charles's Law

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \quad (P = \text{constant})$$

$V = bT \qquad \frac{V_1}{T_1} = b$

$\frac{V_1}{T_1} = \frac{V_2}{T_2} \qquad \frac{V_2}{T_2} = b$



Avogadro's Law

For a gas at constant temperature and pressure, the volume is directly proportional to the number of moles of gas (at low pressures).

$$V = an$$

a = proportionality constant

V = volume of the gas

n = number of moles of gas

$P = an$

$P \quad V \quad T \quad n$

Ideal Gas Law

- An equation of state for a gas.
- "state" is the condition of the gas at a given time.

$$PV = nRT$$

$\frac{P_1V_1}{n_1T_1} = R = \frac{P_2V_2}{n_2T_2} \qquad R = 0.08206 \frac{L\cdot atm}{K\cdot mol}$

$\frac{P_1V_1}{\cancel{P_1}\cancel{T_1}} = \frac{P_2V_2}{\cancel{P_2}\cancel{T_2}}$ constant n & T

$\frac{V_1}{T_1} = \frac{V_2}{T_2}$ n & P

$\frac{V_1}{n_1} = \frac{V_2}{n_2}$ P & T

5.20 What is the effect of the following on the volume of 1 mol of an ideal gas?

(a) The pressure is tripled (at constant T).

(b) The absolute temperature is increased by a factor of 3.0 (at constant P).

(c) Three more moles of the gas are added (at constant P and T).

a.) $\frac{P_1V_1}{\cancel{n_1}\cancel{T_1}} = \frac{P_2V_2}{\cancel{n_2}\cancel{T_2}}$

$P_1V_1 = P_2V_2$

$PV_1 = 3PV_2$

c.) $\frac{V_1}{n_1} = \frac{V_2}{4}$

$V_2 = 4V_1$

5.24 A weather balloon is filled with helium to a volume of 1.61 L at 734 torr. What is the volume of the balloon after it has been released and its pressure has dropped to 0.844 atm? Assume that the temperature remains constant.

$V_1 = 1.6\,L$ $\quad V_2 = ?$

$P_1 = 734$ torr $\quad P_2 = 0.844$ atm

$$\frac{P_1V_1}{\cancel{n_1}\cancel{T_1}} = \frac{P_2V_2}{\cancel{n_2}\cancel{T_2}}$$

$$V_2 = \frac{P_1V_1}{P_2} = \frac{(1.6L)(734\text{ torr})}{641\text{ torr}} = 1.84\,L$$

$$0.844\text{ atm} \times \frac{760\text{ torr}}{1\text{ atm}} = 641\text{ torr}$$

760 torr = 760 mmHg = 1 atm

5.28 A gas cylinder is filled with argon at a pressure of 177 atm and 25°C. What is the gas pressure when the temperature of the cylinder and its contents are heated to 195°C by exposure to fire?

$P \propto T$ $\quad n_{const}$ $\quad V_{const}$ $\quad P_1 = 177$ atm $\quad T_1 = 25°C$

$PV = nRT$

$$\frac{P_1\cancel{V_1}}{\cancel{n}T_1} = R = \frac{P_2\cancel{V_2}}{\cancel{n_2}T_2} \qquad \frac{P_1}{T_1} = \frac{P_2}{T_2} \qquad T_2 = 195°C \quad P_2 = ?$$

$$P_2 = \frac{P_1T_2}{T_1} = \frac{(177\text{ atm})(\overset{195+273K}{\cancel{298.15K}})}{(298.15K)} \qquad P_2 = 278\text{ atm}$$

Standard Temperature and Pressure

"STP"

P = 1 atmosphere

T = 0°C $\quad$ 273.15 K

The molar volume of an ideal gas is 22.42 liters at STP

Ideal Gas Law

$$PV = nRT$$

moles

R = proportionality constant

= 0.08206 L atm K^{-1} mol^{-1}

P = pressure in atm

V = volume in liters

n = moles

T = temperature in Kelvins

Holds closely at $P < 1$ atm

5.32 A sample of Freon-12 (CF_2Cl_2) occupies 25.5 L at 298 K and 153.3 kPa. Find its volume at STP.

$PV = nRT$

$$V_{stp} = \frac{nRT}{P}$$

$T_1 = 298K \quad T$

$P_1 = 153.3\,kPa \times \frac{1\text{ atm}}{101.325\,kPa} = 1.51$ atm

$V_1 = 25.5\,L$

$R = 0.08206 \frac{L\cdot atm}{K\cdot mol}$

$$n = \frac{P_1V_1}{RT_1} = \frac{(1.51\text{ atm})(25.5L)}{(0.08206 \frac{L\cdot atm}{K\cdot mol})(298K)} = 1.57\text{ mol}$$

$$V_{stp} = \frac{(1.57\text{ mol})(0.08206\frac{L\cdot atm}{K\cdot mol})(273.15K)}{1\text{ atm}} = 35.2L$$

$$\frac{P_1V_1}{\cancel{n}T_1} = R = \frac{P_2V_2}{\cancel{n}T_2}$$

5.34 A sample of chlorine gas is confined in a 5.0-L container at 328 torr and 37°C. How many moles of gas are in the sample?

$PV = nRT$

$P = 328\text{ torr} \times \frac{1\text{ atm}}{760\text{ torr}} = 0.432$

$T = 37°C$

$V = 5.0L$

$$n = \frac{PV}{RT} = \frac{(0.432\text{ atm})(5.0L)}{(0.08206\frac{L\cdot atm}{K\cdot mol})(273+37\,K)} \qquad n = ?$$

$= 0.085$ mol Cl_2

5.36 You have 357 mL of chlorine trifluoride gas at 699 mmHg and 45°C. What is the mass (in g) of the sample?

$PV = nRT$

$P = 699\ mmHg$

$V = 357\ mL$

$T = 45\ °C$

$$n = \frac{PV}{RT} = \frac{(0.920\ atm)(0.357\ L)}{\left(0.08206\ \frac{L\cdot atm}{K\cdot mol}\right)(318.15\ K)}$$

$= 0.0126\ mol\ ClF_3$

$$0.0126\ mol\ ClF_3 \times \frac{92.418\ g}{1\ mol\ ClF_3} = 1.20\ g\ ClF_3$$

5.45 What is the density of Xe gas at STP?

$PV = nRT$

$d = \frac{m}{V}$

$MW = \frac{m}{n}$

$n = \frac{m}{(MW)}$

$PV = \frac{mRT}{(MW)}$

$d = \frac{m}{V} = \frac{P(MW)}{RT}$

He 4.0 amu

.8 N_2 .2 O_2 29 amu

$$d = \frac{(1\ atm)(131.3\ g/mol)}{\left(0.08206\ \frac{L\cdot atm}{K\cdot mol}\right)(273\ K)} = 5.86\ \frac{g}{L}$$

5.50 When an evacuated 63.8-mL glass bulb is filled with a gas at 22°C and 747 mmHg, the bulb gains 0.103 g in mass. Is the gas N_2, Ne, or Ar?

$PV = nRT$

$$n = \frac{PV}{RT} = 0.00259\ mol$$

$$MW = \frac{0.103\ g}{0.00259\ mol} = 39.768 \quad Ar$$

5.55 How many grams of phosphine (PH_3) can form when 37.5 g of phosphorus and 83.0 L of hydrogen gas react at STP?

$P_4(s) + 6H_2(g) \longrightarrow 4PH_3(g)$ [unbalanced]

x y ?

mol mol

limiting

$$0.3027\ mol\ P_4 \times \frac{6\ mol\ H_2}{1\ mol\ P_4} = 1.816\ mol\ H_2$$

3.7029 mol H_2

$PV = nRT$

$$n = \frac{PV}{RT} = \frac{(1\ atm)(83\ L)}{(0.08206)(273)} =$$

$$0.3027\ mol\ P_4 \times \frac{4\ mol\ PH_3}{1\ mol\ P_4} \times \frac{33.998\ g\ PH_3}{1\ mol\ PH_3} = 41.2\ g\ PH_3$$

Dalton's Law of Partial Pressures

$1\ atm = P_T = P_{air} + P_{H_2O}$

For a mixture of gases in a container,

$$P_{Total} = P_1 + P_2 + P_3 + \ldots$$

5.57 Aluminum reacts with excess hydrochloric acid to form aqueous aluminum chloride and 35.8 mL of hydrogen gas over water at 27°C and 751 mmHg. How many grams of aluminum reacted?

P_{H_2O} = 26.7 torr at 27 °C

$$2Al_{(s)} + 6HCl_{(aq)} \rightarrow 2AlCl_{3(aq)} + 3H_{2(g)}$$

g=?

2.49×10^{-3} #1

mol H_2

35.8 mL

$P_T = P_{H_2} + P_{H_2O}$

$PV = nRT$

$P_{H_2} = P_T - P_{H_2O}$

$n_{H_2} = \frac{P_{H_2}V}{RT}$

$= 751\ mmHg - 26.7\ mmHg = 724.3\ mmHg$

TABLE B5.1	Composition of Clean, Dry Air at Sea Level
Component	**Mole Fraction**
Nitrogen (N_2)	0.78084
Oxygen (O_2)	0.20946
Argon (Ar)	0.00934
Carbon dioxide (CO_2)	0.00040
Neon (Ne)	1.818×10^{-5}
Helium (He)	5.24×10^{-6}
Methane (CH_4)	2×10^{-6}
Krypton (Kr)	1.14×10^{-6}
Hydrogen (H_2)	5×10^{-7}
Dinitrogen monoxide (N_2O)	5×10^{-7}
Carbon monoxide (CO)	1×10^{-7}
Xenon (Xe)	8×10^{-8}
Ozone (O_3)	2×10^{-8}
Ammonia (NH_3)	6×10^{-9}
Nitrogen dioxide (NO_2)	6×10^{-9}
Nitrogen monoxide (NO)	6×10^{-10}
Sulfur dioxide (SO_2)	2×10^{-10}
Hydrogen sulfide (H_2S)	2×10^{-10}

$P_{CO_2} = \chi_{CO_2} P_T$

$= 0.00040 (1\,atm) = 4.0\times10^{-4}\,atm$

$\frac{400\,mol\,CO_2}{10^6\,mol\,air}$

$CO_2 = 400\,ppmv$ ppb ppt $\quad N_2 = \frac{0.78}{1} = \frac{78}{100}$

$SO_2 \quad 2\times10^{-10} \times 10^9 = 0.20\,ppb$

Partial Pressure

The partial pressure of a gas equals the mole fraction of the gas time the total pressure.

$$P_A = \chi_A P_{Total}$$

$\chi_A = \frac{mole\ A}{total\ mol} = \frac{vol\ A}{total\ \cancel{A}}$

Carbon Dioxide, an important greenhouse gas, is currently measured at 400 ppm in the atmosphere. How many molecules of carbon dioxide would there be in 1.0 L of air at STP.

$P_{CO_2} = \chi_{CO_2} P_T \qquad PV = nRT$

$n_{CO_2} = \frac{P_{CO_2} V}{RT}$

$P_{CO_2} = \left(\frac{400\,mol\,CO_2}{10^6\,mol\,Air}\right) 1\,atm$

$P_{CO_2} = 4.00\times10^{-4}\,atm$

$n_{CO_2} = \frac{(4.00\times10^{-4}\,atm)(1.0\,L)}{0.08206\ (273\,K)}$

$= 1.785\times10^{-5}\,mol\,CO_2$

$\times \frac{6.022\times10^{23}\,molec\,CO_2}{1\,mol\,CO_2} = 1.075\times10^{19}\,molec\,CO_2$

Kinetic Molecular Theory

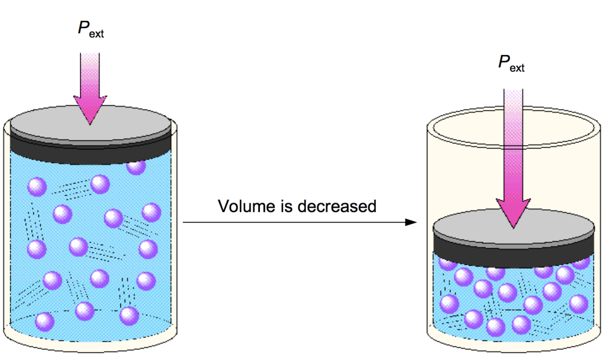
1. Volume of individual particles is ≈ zero.
2. Collisions of particles with container walls cause pressure exerted by gas.
3. Particles exert no forces on each other.
4. Average kinetic energy ∝ Kelvin temperature of a gas.

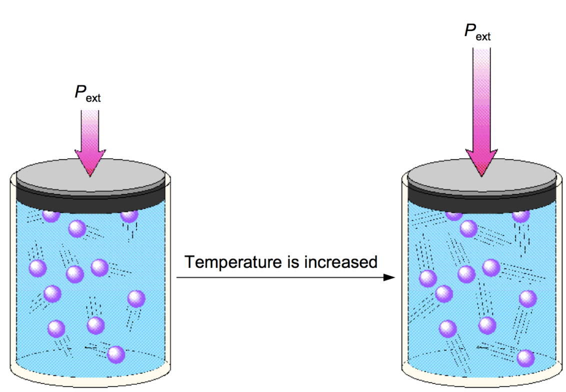
Ideal Gas $\quad$ low P $\quad$ High T

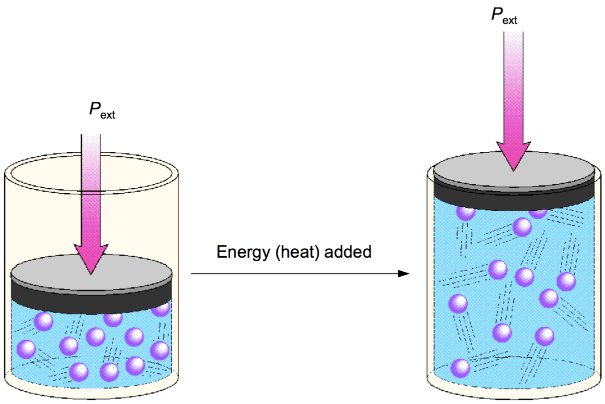
He

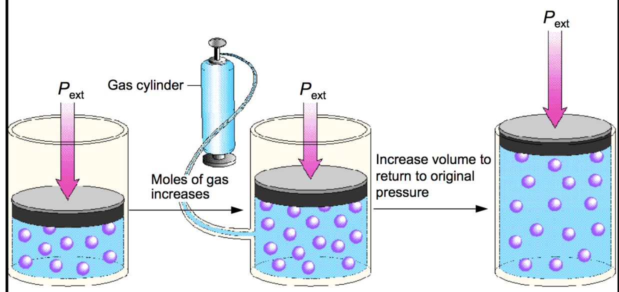
Ar

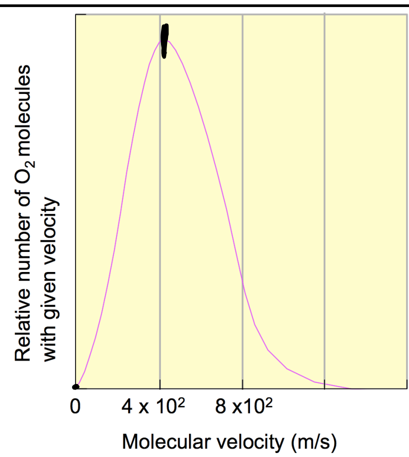
$T\uparrow \quad \uparrow KE = \frac{1}{2} m v^2$

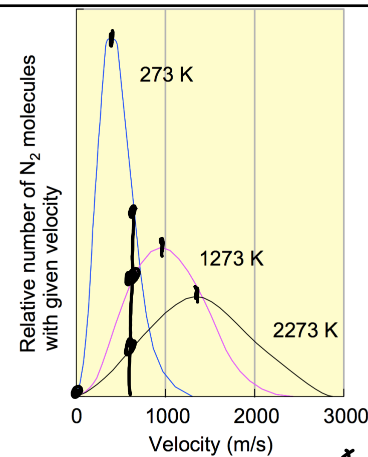










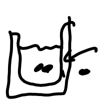


At T↑ KE↑

The Meaning of Temperature

$$(\mathrm{KE})_{\mathrm{avg}} = \frac{3}{2} RT$$

Kelvin temperature is an index of the random motions of gas particles (higher T means greater motion.)

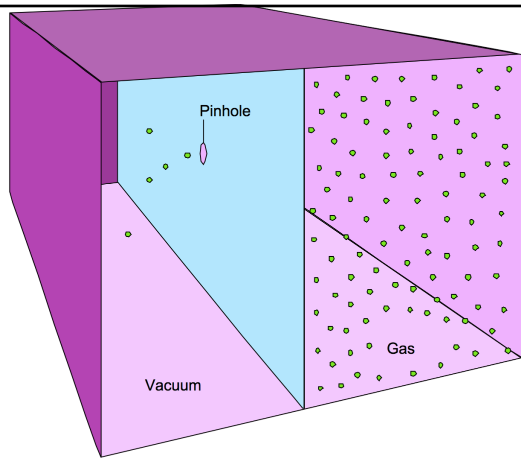


VAC.

Diffusion: describes the mixing of gases. The rate of diffusion is the rate of gas mixing.

Effusion: describes the passage of gas into an evacuated chamber.

He Ar



Effusion:

$$\frac{\text{Rate of effusion for gas 1}}{\text{Rate of effusion for gas 2}} = \frac{\sqrt{M_2}}{\sqrt{M_1}}$$

Diffusion:

$$\frac{\text{Distance traveled by gas 1}}{\text{Distance traveled by gas 2}} = \frac{\sqrt{M_2}}{\sqrt{M_1}}$$

Real Gases

Must correct ideal gas behavior when at high pressure (smaller volume) and low temperature (attractive forces become important).

Real Gases

$$\overset{P}{\underbrace{[P_{obs} + a(n/V)^2]}_{\substack{\uparrow \\ \text{corrected pressure} \\ P_{ideal}}}} \times \overset{V}{\underbrace{(V - nb)}_{\substack{\uparrow \\ \text{corrected volume} \\ V_{ideal}}}} = nRT \quad = nRT$$

END