Handout 11: Understanding Probabilities Associated with Medical Screening Tests
STAT 100 – Spring 2016

Example: Using Mammograms to Screen for Breast Cancer

Gerd Gigerenzer, a German psychologist, has conducted several studies to investigate physicians' understanding of health statistics (Gigerenzer 2010). In one study, he asked 160 gynecologists about the following routine event:

A woman tests positive in a mammography screening. She wants to know what her chances are that she actually has breast cancer. You know the following information about women in this region:

- *About 1% of the women in this region have breast cancer*
- *If a woman has breast cancer, there is a 90% chance that the mammogram will correctly tell her she has breast cancer*
- *If a woman does not have breast cancer, there is a 9% chance that the mammogram will incorrectly tell her she has breast cancer (this is called a false positive)*

If a woman tests positive, what is the best estimate for the probability that she has breast cancer: 90%, 81%, 10%, or 1%?

Gigerenzer's results are summarized in the table below.

Answer	Percent that chose this Answer
90%	47%
81%	13%
10%	21%
1%	19%

Questions:

1. How do you feel about the variability that was observed in the physicians' statistical thinking?

   Answers will vary.

2. Which answer do you think is correct, and why? How confident are you in your answer?

   Answers will vary.

To find the correct answer, consider the following. Suppose that 10,000 women participate in the mammography screening test for breast cancer. Remember what we know about the women in this region:

- About 1% of the women in this region have breast cancer
- If a woman has breast cancer, there is a 90% chance that the mammogram will correctly tell her she has breast cancer
- If a woman does not have breast cancer, there is a 9% chance that the mammogram will incorrectly tell her she has breast cancer

Translate this information into the appropriate counts on the table below.

	Test Positive	Test Negative	*Total*
Cancer	**90**	**10**	**100**
No cancer	**891**	**9,009**	**9,900**
Total	**981**	**9,019**	**10,000**

Questions:

1. Given that a woman tests positive for breast cancer according to her mammogram, what is the probability that she has breast cancer? Use the above table of counts to find this probability.

   90/981 = 9.2%

2. Was your intuition correct on this? If not, are you surprised by this result?

   Answers will vary.

In the previous study, many of the physicians confused these two probabilities:

- The probability of a positive test result *given that a woman has breast cancer* is 90%
- The probability of having breast cancer *given a positive test result* is about 9.2%

The correct answer depends on which conditional probability is really of interest (i.e., it depends on what event we're assuming is "given" when calculating the probability of interest). To answer the question, "*What are the chances I have breast cancer if my mammogram tests positive?*" we must condition on getting a positive test.

This is a mistake that is often made by not only physicians but also by members of the general public – we're often confused when the results associated with screening tests are reported in terms of probabilities. Gigerenzer argues that we should always construct a table of counts (as shown above) when examining the effectiveness of screening tests. In the remainder of this handout, you will look at a few more examples of screening tests using Gigerenzer's advice. You'll also be introduced to some of the formal terminology that is typically associated with screening tests.

Definitions

- The **prevalence** of a disease can be viewed as the proportion of a population that has that disease
- The **sensitivity** of a test is the probability that test results in a true positive (i.e., the probability the result is positive given that the subject has the disease)
- The **specificity** of a test is the probability that test results in a true negative (i.e., the probability the result is negative given that the subject does not have the disease)
- The **false positive rate** of a test is the probability that the test results in a false positive (i.e., the probability the result is positive given that the subject does not have the disease)
- The **false negative rate** of a test is the probability that the test results in a false negative (i.e., the probability the result is negative given that the subject has the disease)
- The **positive predictive value** of a test is the probability that a subject actually has the disease given that they tested positive for it.

- The **negative predictive value** of a test is the probability that a subject actually does not have the disease given that they tested negative for it.

Questions: Find the following for the Mammogram/Breast Cancer example. It may help to sketch in your results from page 2 on the following table of counts.

	Test Positive	Test Negative	*Total*
Cancer	90	10	100
No cancer	891	9,009	9,900
Total	981	9,019	10,000

1. Find the prevalence of the test.

   100/10,000 = 1%

2. Find the sensitivity of the test.

   90/100 = 90%

3. Find the specificity of the test.

   9,009/9,900 = 91%

4. Find false positive rate of the test.

   891/9900 = 9%

5. Find false negative rate of the test.
   10/100 = 10%

6. Find the positive predictive value of the test.
   90/981 = 9.2%

7. Find the negative predictive value of the test.
   9,009/9,019 = 99.9%

Example: Screening Tests for HIV

Enzyme Immunoassays (EIAs) are commonly used to screen for HIV. According to the CDC, these tests are believed to have an extremely high sensitivity (99.9%) and specificity (99.8%). The CDC also estimates the prevalence of HIV to be about .4% in the general population of the U.S.

Questions:

1. Consider a hypothetical group of 1,000,000 U.S. citizens that are tested for HIV using an EIA test. Use the information from above to fill in the following table of counts.

	Test Positive	Test Negative	*Total*
Has HIV	**3,996**	**4**	**4,000**
Does not have HIV	**1,992**	**994,008**	**996,000**
Total	**5,988**	**994,012**	**1,000,000**

2. Suppose that someone tests positive for HIV using an EIA test. What is the probability that this person actually has HIV given that they tested positive? In other words, find the positive predictive value of this test. Note that you should be using the above table of counts to find this probability.

   3,996 / 5,988 = 66.7%

3. Suppose that someone tests negative for HIV using an EIA test. What is the probability that this person is actually HIV-free given that they tested negative? In other words, find the negative predictive value of this test. Note that you should be using the above table of counts to find this probability.

   994,008 / 994, 012 = 99.9996%

It is well-documented that the prevalence of HIV is higher in certain groups of people than in the general population of U.S. citizens. Suppose that the prevalence of HIV in a certain population of interest is 2%. Once again, assume that an EIA test has a sensitivity of 99.9% and a specificity of 99.8%.

Questions:

1. Consider a hypothetical group of 1,000,000 people from this specific population that are tested for HIV. Use the information from above to fill in the following table of counts.

	Test Positive	Test Negative	*Total*
Has HIV	**19,980**	**20**	**20,000**
Does not have HIV	**1,960**	**978,040**	**980,000**
Total	**21,940**	**978,060**	**1,000,000**

2. Suppose that someone in this certain population tests positive for HIV using an EIA test. What is the probability that this person actually has HIV given that they tested positive? In other words, find the positive predictive value of this test. Note that you should be using the above table of counts to find this probability.

   19,980 / 21,940 = 91%

3. Suppose that someone in this certain population tests negative for HIV using an EIA test. What is the probability that this person is actually HIV-free given that they tested negative? In other words, find the negative predictive value of this test. Note that you should be using the above table of counts to find this probability.

   978,040 / 978,060 = 99.998%

4. What happened to the positive predictive value of the test as the prevalence of the disease increased? Hint: compare your answers to Question 2 on page 5 and Question 2 on page 6.

   The positive predictive value is higher when the prevalence of the disease is higher.

Finally, note the following information provided by the CDC regarding the use of HIV tests. Hopefully, this information is much easier to understand after working through this handout. *(Source: http://www.cdc.gov/hiv/testing/lab/clia/rtcounseling.html)*

How Do Rapid HIV Tests Compare with Standard HIV Screening Tests, Enzyme Immunoassays (EIAs)?

Clinical studies have demonstrated that the sensitivity[2] and the specificity[3] of rapid HIV tests are comparable to those of EIAs often used for screening. The negative predictive value4 of a screening test is high at the HIV prevalence observed in most U.S. testing settings (CDC, 1998). Therefore, a client with a negative rapid HIV test result can be told he or she is not infected. However, because HIV antibodies take time to develop, retesting should be recommended to persons with a recent possible exposure (sexual contact or needle sharing within 3 months). As with any screening test, the positive predictive value of a reactive rapid HIV test may be low in populations with low prevalence (see Appendix). Because some reactive test results may be false-positive, every reactive rapid test must be confirmed by a supplemental test (either Western blot or immunofluorescence assay [IFA]). (CDC, 1989).

Appendix: Positive Predictive Value of Rapid HIV Tests

Positive predictive value is an important concept that may be difficult to understand. It depends both on the test that is used (in particular, the test's specificity) and the prevalence of infection in the population tested. An example may help to illustrate how the positive predictive value (and the proportion of false-positive test results) changes at different levels of prevalence.

We will illustrate a test that has a sensitivity of 99.9% and a specificity of 99.8%, similar to that of many rapid HIV tests and EIAs. A specificity of 99.8% means that 0.2% (2 tests out of 1,000) will be false-positive. For this example, we will test 1,000 persons, first in an STD clinic with high HIV prevalence: 5%. Testing 1,000 persons, we would discover 50 persons who were truly positive. Based on the test's specificity, we would also encounter 2 false-positive test results. Thus, the positive predictive value of a reactive test in this setting would be (50 true positive tests) divided by (52 total positive tests) or 96%.

Using this same test in a population with low prevalence gives us a very different predictive value. For this example, we will use the same test in a family planning clinic, where the HIV prevalence is 0.1%. Testing 1,000 persons in this clinic, 1 person would be truly positive, but again, 2 test results would be false-positive. The positive predictive value of a reactive test in this setting, therefore, would be (1 true positive test) divided by (3 total positive tests) or 33%. Notice that in both these examples, the ***number*** of false-positive tests is the same, but the ***proportion*** of false-positive tests is very different.

The following table shows the positive predictive values at different levels of HIV prevalence for a test with 99.8% specificity.

Positive Predictive Value of HIV Tests in Populations with Differing HIV Prevalence
Example: Testing 1,000 Persons

HIV Prevalence	True Positive (Number)	False Positive (Number)	Positive Predictive Value
10%	100	2	98%
5%	50	2	96%
2%	20	2	91%
1%	10	2	83%
0.5%	5	2	71%
0.2%	2	2	50%
0.1%	1	2	33%