STAT 110: Homework 3 Solutions (20 pts)
Fall 2017

1. Consider Problem 2 from the previous homework assignment. A recent opinion poll surveyed 140 people who were randomly selected from a particular town and found that 90 of them disapproved of the way the mayor of that town was doing his job.

    a. Find a 95% confidence interval for the true proportion of this town's population who disapprove of their mayor's job performance. Be sure to report the sample statistic, the margin of error, and the lower/upper endpoints of the confidence interval. (3 pts)

    Sample statistic:

    $\hat{\pi} = 90/140 = 0.64$

    Margin of error:

    $$1.96\sqrt{\frac{0.64 \times (1 - 0.64)}{140}} = 0.08$$

    Lower endpoint: <u>**0.64 – 0.08 = 0.56**</u>

    Upper endpoint: <u>**0.64 + 0.08 = 0.72**</u>

    b. Interpret this confidence interval <u>in the context of the problem</u> using everyday language. (2 pts)

    We can be 95% certain that the true proportion of this town's population who disapprove of the mayor's job performance is captured in the interval (0.56, 0.72). In other words, we are 95% certain that the true proportion of this town's population who disapprove of the mayor's job performance is somewhere between 56% and 72%.

2. A random sample of 200 third-graders in a large school district was chosen to participate in an after-school reading program to improve reading skills. After completing the program, the children were tested, and 142 of them showed improvement in their reading skills.

a. Find a point estimate for the proportion of all third-graders in this school district whose reading scores would improve after completing the program. (1 pt)

$\hat{\pi} = 142/200 = 0.71$

b. Construct a 95% confidence interval for the proportion of all third-graders in this school district whose reading scores would improve after completing the program. (2 pts)

Margin of error:

$$1.96 \sqrt{\frac{0.71 \times (1 - 0.71)}{200}} = 0.06$$

Lower endpoint: **0.71 – 0.06 = 0.65**

Upper endpoint: **0.71 + 0.06 = 0.77**

c. Is it reasonable to conclude that more than 60% of all third-graders in this school district would improve their reading scores after completing the program? Explain. **Use the confidence interval to explain your reasoning (i.e., don't find a p-value).** (1 pt)

The confidence interval tells us that we can be 95% certain that the true proportion of all third-graders in this school district whose reading scores would improve after completing the program is between 65% and 77%. Since all the values in this interval are greater than 60%, we have evidence that the true proportion whose reading scores would improve is greater than 60%.

3. A study was conducted to investigate what patients were more likely to value when choosing a doctor: (1) interpersonal skills, or (2) technical ability. In this study, 304 people were asked to choose a physician based on two hypothetical descriptions. One physician was described as having high technical ability and average interpersonal skills, and the other was described as having average technical ability and high interpersonal skills. The physician with high technical ability was chosen by 188 of the people surveyed. **The research hypothesis was that the majority of all patients would prefer the physician with high technical ability**.

a. Set up the null and alternative hypotheses to investigate this. (1 pt)

Let π = the probability a patient would prefer the physician with high technical ability (i.e., the proportion of all patients who would prefer the physician with high technical ability).

H$_o$: π = 50% **(there is no preference for one physician or the other)**

H$_a$: π > 50% **(there is a preference for the physician with high technical ability)**

b. Calculate the statistic of interest in this study. (1 pt)

$\hat{\pi}$ = 188/304 = 0.62

c. Use the binomial distribution to find the exact p-value. State the values of n, π, and the p-value below. (2 pts)

n = **304**

π = **0.50**

Exact p-value = **2.16 x 10-5 = 0.0000216**
(the probability of observing 188 or more out of 304 assuming π = 0.50)

d. Write a conclusion <u>in the context of the problem</u> to address whether the evidence supports the research hypothesis. (2 pts)

The study provides evidence that the majority of all patients would prefer the physician with high technical ability.

4. A group of subjects was randomly selected to participate in a study investigating dishonesty. They were told that if they filled out a survey for the researchers, they would be paid $20 for their time. After the survey was complete, they were given a six-sided die and were told they might receive an additional payoff, depending on the outcome of a die roll. If they rolled a 1, 2, 3, 4, or 5, they would get an additional $5. If they rolled a 6, however, their additional payoff was $0. They were told that only the first toss of the die was relevant for payoff, but they were encouraged to roll the die more than once to convince themselves it was a fair die. The experimenter did not watch the subjects toss the die, so there was no way for them to know for sure if a given subject lied about the outcome of the first die roll. A total of 389 subjects were included in the study; 25 people reported rolling a 6 on their first toss (which meant they didn't get the additional $5).

    The research hypothesis was that people would tend to be dishonest and report an outcome that would result in the additional $5 payoff, even if they didn't "earn" it.

    a. Set up the null and alternative hypotheses to investigate this. (1 pt)

        Let π = the proportion of people who would report rolling a six on their first toss of the die.

        H_o: $\pi = 1/6$ **(people are honest, so sixes will appear 1/6 of the time)**

        H_a: $\pi < 1/6$ **(people are dishonest and tend to report fewer sixes than expected by chance)**

    b. Use the binomial distribution to find the exact p-value for testing this research hypothesis. State the values of n, π, and the p-value below. (2 pts)

        n = **389**

        π = **1/6**

        Exact p-value = **1.3 x 10^{-9}**
        (the probability of observing 25 or fewer out of 389 assuming $\pi = 1/6$)

    c. Write a conclusion <u>in the context of the problem</u> to address whether the evidence supports the research hypothesis. (2 pts)

        The study provides evidence that people tend to be dishonest and report an outcome that would result in the additional $5 payoff, even if they didn't "earn" it.