**STAT 110: What Does 95% Confidence Really Mean for a Confidence Interval?**
**Fall 2017**

Once again, consider the *Star Tribune* poll. Note that if we were to obtain another random sample of 800 Minnesotans, the sample proportion from this new study will probably *not* be equal to 408/800 = 51%. Thus, the confidence interval we calculate based on this statistic will also change.

Also, note that the population parameter ($\pi$ = the proportion of all Minnesotan adults who support legalization of medical marijuana) is equal to one true value, which happens to be unknown. We have no way of knowing *for sure* whether the confidence interval obtained from the single *Star Tribune* poll has captured this one true value.

We can, however, conduct a simulation study to see how well our methods for constructing a confidence interval work. In this simulation study, we (1) set the population proportion to a value that is known, (2) pretend like we don't know the true population proportion and construct confidence intervals to estimate it from several random samples, and (3) see how often the resulting interval captures the truth.

This process is easy to do using an applet available on the web. Open the "Simulating Confidence Intervals" applet.

Link: http://www.rossmanchance.com/applets/ConfSim.html

Carry out the following steps.

1. Let's assume that the true population proportion is $\pi = 0.50$. In the applet, set the value of $\pi$ to be 0.50 and n to be 800. Also, make sure that the method is set to "Wald" for "Proportions."

2. Click on "Sample." Then, click on the interval (i.e., click on the line that represents the confidence interval on the graph) to see values of the endpoints. Identify the following from the first random sample obtained in your simulation study.

    - Sample proportion $\hat{\pi}$:

    - 95% confidence interval:

    - Did the confidence interval capture the true value of $\pi = 0.50$?

3. Click “Sample” again. Did you get the same interval? Does this interval capture the true value of $\pi = 0.50$?

4. To investigate what happens in the long run, we can use the applet to take many more random samples and construct a confidence interval for each. Change the number of “Intervals” from 1 to 98 and click “Sample.” What percentage of the 100 intervals capture the true value of the population proportion, $\pi = 0.50$? Note that this information is given in the “Running Total.”

5. Change the number of “Intervals” from 98 to 100 and click “Sample” repeatedly until you’ve created 1,000 confidence intervals. What percentage of these 1000 intervals capture the true value of the population proportion, $\pi = 0.50$? This is often referred to as the *coverage rate*.

6. Predict how the coverage rate will change if you were to construct 90% confidence intervals instead of 95% confidence intervals. Then, in the applet, change the confidence level to 90% and click “Recalculate.” Continue to press “Sample” until you have constructed 1,000 intervals. What percentage of these 1000 intervals capture the true value of the population proportion, $\pi = 0.50$?

7. Now, recall that in the actual poll, you get only one sample and thus one confidence interval. Do you know *for sure* whether this one interval captures the truth? If not, how confident are you that the interval contains the true value of the population parameter?

**<u>Meaning of 95% Confidence</u>**

These simulations show us that if we take repeated samples from a population and construct 95% confidence intervals each time, in the long run approximately 95% of our intervals will succeed in capturing the true value of the population proportion, $\pi$. So, when we collect data in the real world and calculate a single 95% confidence interval, we say we are 95% confident (or certain) that this interval captures the truth.

**More on Comparing the Wald and Score Methods**

As mentioned earlier, the Wald method does not perform as well as Wilson's score method under certain circumstances. Use the "Simulating Confidence Intervals" applet to compare the two for the following values of n and $\pi$ (using 95% confidence intervals).

In each case, report the percentage of intervals (out of 1,000) that capture the true parameter value (i.e., the coverage rate).

| **Values for $\pi$ and n** | **Coverage Rate with Wald's Method** | **Coverage Rate with Wilson's Score Method** |
|---|---|---|
| $\pi$ = 0.15 and n = 10 | | |
| $\pi$ = 0.10 and n = 100 | | |
| $\pi$ = 0.50 and n = 10 | | |
| $\pi$ = 0.90 and n = 50 | | |

Write a paragraph below summarizing these results.

Source: This is a modified version of Exploration 4.3 from *Investigating Statistical Concepts, Applications, and Methods* by Chance and Rossman. 2006. Duxbury, Thomson Brooks/Cole.