EXPERIMENT 1 - Uncertainty and Significant Figures: Measurement of Mass and Density

Materials Needed

20 pennies, 3 aspirin tablets, 3 standard weights, 1 unknown metal cube, electronic balance, ruler

Relevant Textbook Reading

Smith, Chapter 1.4-1.5, 1.10

Background

In science, it is of the utmost importance to make and express measurements reliably and consistently. The scientific method absolutely depends on reliable measurements for the formulation and subsequent testing of the theories that are proposed to explain our world. However, measurements, by their very nature, are never exact or completely certain. That is, all measurements contain a certain level of uncertainty. Therefore, it is very desirable that there be a convention for expressing measurements that conveys the amount of uncertainty in the value given. This convention is called the Significant Figures convention.

Numbers and Measurement.

There are two general categories of numbers, exact numbers and measured numbers.

- Exact numbers have no degree of uncertainty present. For example, counted numbers are exact numbers: the number of people in your lab section or the number of fingers on your hand. In addition, defined numbers such as 12 inches per foot, 16 ounces per pound are also exact numbers. To say there are 28.5 people in your lab section is meaningless. A pound equals exactly 16 ounces by definition.

- Measured numbers are the numbers that are determined by an experiment. For examples measured numbers can be the length of a material measured by a yard stick, or the mass of a substance measured by a balance. Measured numbers are always uncertain to a degree.

Of course, when making measurements we try to minimize the amount of uncertainty in our result. The relative certainty of a measurement (how "good" it is) can be looked at in two ways.

- **Accuracy**. If a "true" value is known, then how close your measurement comes to the true value is an indication of how good it is. The level of agreement between a measured value and the true value is called **accuracy**. The accuracy of measurement is often reported in terms of the **Percent Error.** The percent error is simply the difference between the true value and the measured value (i.e., the error) as a percentage of the true value.

$$\% \text{ error} = (\text{measured value} - \text{true value})/\text{true value} \times 100$$

- **Precision**. If the measurement is repeated, how close each of the individual measurements come to each other is also an indication of how good the measurement is. The level of agreement between the results of several trials of the same measurement is called **precision**.

There are two types of uncertainty (also called "experimental error"). These are called systematic error and random error and these affect the value being measured in different ways.

- **Systematic Error** is uncertainty due to miscalibration of the measuring device and/or mistaken assumptions in the calculation method used to obtain a measured value. The hallmark of systematic error is that repeated measurements will always be off in the same direction, i.e., either too high or too low. Therefore, **systematic error mainly affects the accuracy** of a measurement.

- **Random Error** is uncertainty due to the randomness involved in trying to read an analog measuring device as finely as possible. (See the part on measurement and the 10% rule below.) Unlike systematic error, random error has the affect of making repeated measurements randomly too high or too low. Therefore, **random error mainly affects the precision** of a measurement. Also if several trials of the same measurement are performed and the results averaged, then the random error will tend to cancel itself out. Because of this scientists, knowing that random error will be minimized by doing so, often carry out at least three determinations of every measurement and report the average result.

- **Random Error** is also present in the readouts from digital measuring devices but it is disguised by the fact that digital instruments usually do not read out beyond the first digit that is uncertain. Because of this, some people would believe that if a digital balance reads out 1.314 g then that is the "exact" mass of the object. What is missed here is that the 4th decimal place could be any number from 0 to 4. In other words, the actual mass could be anywhere from 1.3140 to 1.3145 grams. (In fact, the actual mass could be anywhere from 1.3135 to 1.3145 grams because in the lower end of this range the 3rd decimal would round up to a 4.) Hence, the 1.314 g measurement should be interpreted as 1.314 $\pm$ 0.001 g. If the same object was weighed on a balance that only read to the nearest tenth it would read as 1.3 g, which would be interpreted as 1.3 $\pm$ 0.1 g. Clearly the balance that reads to the third decimal place has less random error and is, therefore, more precise. **The more significant figures in a measured value, the more precise it can be assumed to be.**

Measurement and the 10% Rule. The number of significant figures (SF) in a measurement always includes one estimated digit when reading the measured value on a calibrated scale. We include one estimated digit because it is best to try to get as much information as possible by reading the value as closely as possible. Therefore, it is standard practice to estimate 0.1 times (or 10%) of the distance between the nearest adjacent calibration marks of the measuring device. The estimated digit represents the last significant figure in the measurement.

Example: Consider the line segment below in relation to the arbitrary measuring scale shown. The ruler is calibrated to 0.1 units thus 10% would be 0.01 units. The uncertainty in a measurement using the ruler shown below is then $\pm$ 0.01 units. We can say for sure that the line segment is 0.3 units long. But as you can see, the segment is closer to 0.31 units long than 0.30 or 0.32 units. Thus, the best way to report the measurement is as 0.31 units, which is properly interpreted by the reader as 0.31 $\pm$ 0.01 units correctly reflecting the fact that the 2nd decimal place was estimated.
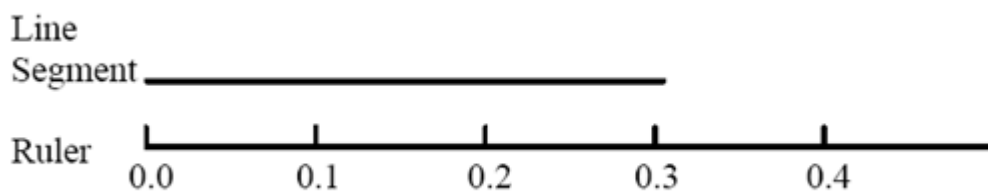
Common Laboratory Measurements:

- **Volume Measurements.** The volume of a sample is the total amount of space occupied by the sample. When cooking, liquid volumes are measured in units of teaspoons, tablespoons and cups. In the laboratory, liquid volumes are typically measured by using graduated cylinders. A graduated cylinder is read by observing the bottom of the meniscus level of the liquid and reading to 0.1 times (the 10% Rule) of the smallest calibrated mark.

- **Mass Measurements.** Mass is measured in the laboratory by using a balance. Most electronic balances can be "tared". To tare a balance means to set the display equal to zero while the container is on the balance. Then the mass of the matter being weighed can be directly read from the balance, without having to subtract the mass of the container.

Significant Figures in Calculations. See Chapter 1.5 in Smith for rules on the amount of significant figures to use in the result of a calculation. In general, the result of a division/multiplication operation on a set of measurements should only be given the same number of significant figures as the measurement with the least significant figures. In an addition/subtraction operation the result is only significant to the same decimal place as the measurement with the least significant decimal places.

The Penny. The Lincoln penny of 1909 commemorated the centennial of Abraham Lincoln's birth. It was the first regular-issue U.S. coin to bear the portrait of an actual American. In 1943, a wartime copper shortage prompted the introduction of a zinc-coated steel penny. The coin, however, proved so unpopular that the U.S. Mint resumed production of the copper cent. Pennies minted in 1944 and 1945 were struck from copper salvaged from spent ammunition cartridges. Since 1959 the Lincoln penny has remained exactly the same in outward appearance. However, there was a change over in the actual metal composition used from almost pure copper (95% Cu/5% Zn alloy) to mostly zinc with a thin copper plating. Your measurements in this lab will allow you to discover the year the changeover was made.

Procedures

1. Mass of Lincoln Pennies. Weigh each of the 20 provided Lincoln pennies using a balance that reads to the nearest milligram (a "milligram balance").

2. Mass of aspirin tablets. Weigh three aspirin tablets using a milligram balance. Record the masses using milligrams as the unit.

3. Mass of standard weights. There will be sets of standard weights (usually used to check the calibration of a balance) available in the lab. Choose three different weights (two light ones and one heavier one) and determine the mass of each using both a milligram balance (balance 1) and the home kitchen balance (balance 2) available in the lab.

4. Density of Unknown Metal. Determine the mass of the provided metal cube using a milligram balance. Determine its volume by measuring the length of one of the edges in cm and cubing it (Volume = length^3). Determine the density by simply dividing the two measurements (density = mass/volume).

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IN-LAB OBSERVATIONS/DATA

Names_____ Section _____ Date _____

Make sure to use the proper number or significant figures for all data recorded! Trailing zeros are significant so make sure to record all digits displayed by the electronic balance.

1.   Masses of pennies

Year	Mass (g)	Year	Mass (g)	Year	Mass (g)	Year	Mass (g)

Procedure for above measurements_____

_____

Observations _____

_____

2.   Masses of Aspirin Tablets

Tablet	Mass (mg)
1	
2	
3	

Procedure for above measurements_____

_____

Observations_____

_____

4.   Masses of Standard Weights

Stated Mass (g)	Measured Mass Balance 1 (g)	Measured Mass Balance 2 (g)

Procedure for above measurements_____

_____

Observations_____

_____

5.   Density measurements for unknown metal cube   Unknown # used _____

Trial	Edge length (cm)	Mass (g)
1		
2		
3		

Procedure for above measurements_____

_____

Observations_____

_____

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REPORT

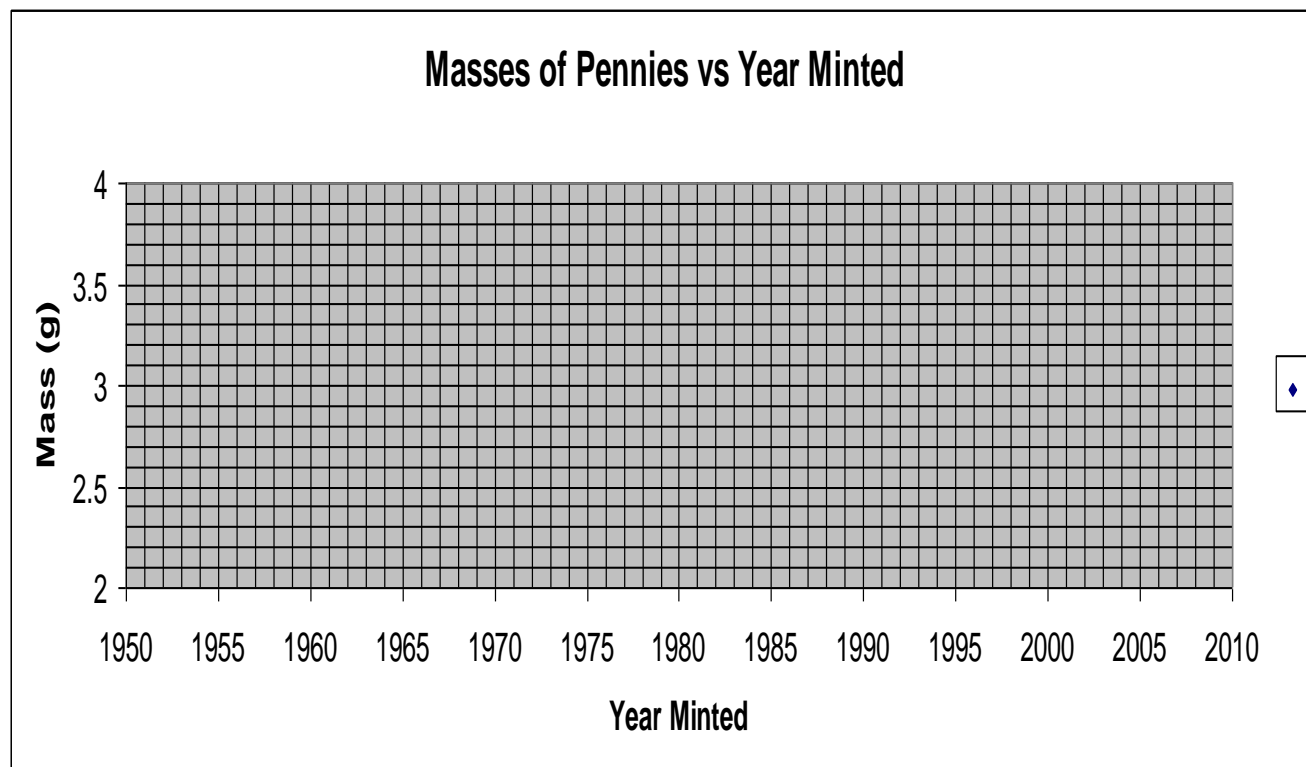**Names**_____ **Section** _____ **Date** _____

Results

1. Density of Unknown Metal   Unknown Number _____

Trial	Mass (g)	Volume (cm^3)	Density (g/cm^3)
1			
2			
3			
Average			

2. Graph of Penny Results

3. Aspirin Tablets

    Average Mass _____   Average Deviation _____

Calculate the average deviation by first determining the "deviation" of each individual measurement from the average. (Subtract the value of the measurement from the average.) Then average the absolute values of the deviations.

4. Standard Weights

Stated Mass (g)	Balance 1		Balance 2	
	mass (g)	% error	mass (g)	% error

Questions (Type up your answers to these on a separate sheet and staple it to this report sheet. Make sure to provide complete answers and use proper grammar and complete sentences.)

1.   The unknown metal cube was one of the following: aluminum, zinc, lead. Compare the known densities of each of these metals (do some research and cite your source) to your density measurement and draw a conclusion as to the identity of your unknown.

2.   Discuss the accuracy and precision of your density measurements. Calculate the percent error of the average and use it to judge the accuracy. Look at how close the results are between the individual trials as an indication of your precision. Also look at the number of significant figures in your calculated densities as an indication of precision.

3.   Identify one specific source of experimental error in the density measurements and classify it as either systematic or random error.

4.   Use your graph of the penny mass data to determine the year when the US Mint changed the composition of the metal used to mint the penny. Also go back to your original data and determine the average mass of a penny before the composition changed and the average mass afterwards.

5.   The aspirin tablets weighed in lab are claimed by the manufacturer to contain 325 mg of aspirin per tablet. What percentage of the tablets consists of other (inactive) ingredients?

7.   Which balance used for the standard weights measurements was more accurate? Which was more precise? Explain fully.

8.   Are instruments with digital readouts necessarily more precise than analog instruments? Explain.