

Parsimonious Modeling and Forecasting of Corporate Yield Curve

WEI-CHOUN YU* AND DONALD M. SALYARDS

*Economics and Finance Department, Winona State University,
Winona, Minnesota, USA*

ABSTRACT

This paper investigates the sensitivity of out-of-sample forecasting performance over a span of different parameters of λ in the dynamic Nelson–Siegel three-factor AR(1) model. First, we find that the ad hoc selection of λ is not optimal. Second, we find a substantial difference in factor dynamics between investment-grade and speculative-grade corporate bonds from 1994:12 to 2006:4. Third, we suggest that the three-factor model is sufficient to explain the main variations of corporate yield changes. Finally, the parsimonious Nelson–Siegel three-factor AR(1) model remains competitive in the out-of-sample forecasting of corporate yields. Copyright © 2008 John Wiley & Sons, Ltd.

KEY WORDS corporate yield curve; Nelson–Siegel model; three-factor model; out-of-sample

INTRODUCTION

Over the past decades, many financial institutions and academics have devoted considerable resources to the modeling of corporate bonds. Most authors focus on explaining the special features of corporate bonds not found in Treasury bonds, such as default probabilities, recovery rates, tax premium, risk premium, and callable properties. However, little attention has been paid to the modeling and forecasting of corporate yield curves from a time-series perspective (especially out-of-sample forecasting), even though forecasting the term structure of corporate interest rates plays a crucial role in portfolio management, household and business finance decisions, and derivative pricing and hedging.

There are two main strands of bond modeling, each with their own focus. First, the no-arbitrage models (Ho and Lee, 1986; Hull and White, 1990) only specify on term-structure cross-sectional fitting. Second, the affine equilibrium models (Vasicek, 1977; Cox *et al.*, 1985; Duffie and Kan, 1996; Dai and Singleton, 2000) only pay attention to instantaneous short rates and result in poor forecasts (Duffee, 2002). In addition to these two main strands, Diebold and Li (2006, hereafter DL) and Diebold *et al.* (2006, hereafter DRA) offer the alternative models concentrating on term-structure

*Correspondence to: Wei-Choun Yu, Economics and Finance Department, Winona State University, Somsen 319E, Winona State University, Winona, MN 55987, USA. E-mail: wyu@winona.edu

forecasting. They extend the parsimonious three-factor (exponential components) yield curve model, proposed by Nelson and Siegel (1987, hereafter NS), to the dynamic form. DL uses a simple two-step approach, in which they first estimate three factors, then model and forecast them. On the other hand, DRA propose a one-step approach that uses the state-space model to simultaneously do factor estimation, modeling, and forecasting. DL's dynamic NS factor autoregressive of order 1 (AR(1)) model produces superior out-of-sample forecasting on US Treasury yields and outperforms the current popular competitors, including the forward rate regression models of Fama and Bliss (1987) and Cochrane and Piazzesi (2005).

Yu and Zivot (2007, hereafter YZ) analyze the pros and cons of DL's and DRA's models. Furthermore, they extend the samples to corporate yields. By and large, they argue that the simple AR(1) method of DL is still the most accurate forecasting model on both Treasury and corporate bonds compared to many other competitors. Nevertheless, in order to compare and evaluate different models' accuracy, DL, DRA and YZ all simplify the set up of the dynamic NS model. They all assume that the λ , which governs the shape and the decay rate of the factor loadings, is fixed. To some extent, they argue, the parameters are not sensitive enough to affect the forecasting results.

In this paper, we investigate the sensitivity of out-of-sample forecasting performance over a span of different parameters of λ in the dynamic NS three-factor AR(1) model. We find that the ad hoc selection of λ is not optimal. Based on the principal component method, we find considerable differences of factor loadings between investment-grade and speculative-grade corporate bonds from 1994:12 to 2006:4. Moreover, we suggest that a three-factor model is sufficient to explain the main variations of corporate yield changes, which is about 92%. The Parsimonious NS three-factor AR(1) model remains competitive in the out-of-sample forecast of corporate yields.

The rest of the paper is organized as follows. The next section introduces the NS model. The third section explores the sensitivity of parameters and evaluates the forecasts. The fourth section investigates unobserved factors by the principal-component method. The fifth section concludes.

MODELS

Nelson–Siegel model

Nelson and Siegel (1987) introduce a parsimonious but influential three-factor model:¹

$$y_t(\tau_i) = \beta_{1t} + \beta_{2t} \left(\frac{1 - e^{-\lambda\tau_i}}{\lambda\tau_i} \right) + \beta_{3t} \left(\frac{1 - e^{-\lambda\tau_i}}{\lambda\tau_i} - e^{-\lambda\tau_i} \right) \quad (1)$$

where τ_i is the maturity of bond i , which usually ranges from 3 months to 30 years. The parameter λ determines the rate of exponential decay. The three factors are β_{1t} , β_{2t} , and β_{3t} . The factor loading on β_{1t} is 1, which is a constant that never dies out. It loads equally at all maturities. In other words, a change in β_{1t} changes all yields uniformly. Therefore, it is called *level factor*. When the maturity becomes larger, β_{1t} plays a more important role in forming yields with respect to smaller factor loadings on β_{2t} and β_{3t} (for instance, $y_t(\infty) = \beta_{1t}$). In consequence, β_{1t} is called long-term factor.

¹The original Nelson–Siegel model is slightly different from equation (1), which is a modified one by DL. DL explains the reasons for this revision.

The factor loading on β_{2t} is $(1 - e^{-\lambda\tau_i})/\lambda\tau_i$, which is a function decaying fast and monotonically to zero. It loads short rates more heavily than long rates; consequently, it changes the slope of the yield curve. β_{2t} is short-term factor, which is also called *slope factor*.² The factor loading on β_{3t} is $(1 - e^{-\lambda\tau_i})/\lambda\tau_i - e^{-\lambda\tau_i}$, which is a function starting at zero (so not short term) and decaying to zero (not long term) with a humped shape in the middle. It loads medium rates more heavily. Accordingly, β_{3t} is the medium-term factor, which is also called *curvature factor* because an increase in β_{3t} will increase the yield curve curvature. Litterman and Scheinkman (1991) use the principal-component method and find the same number and similar pattern of factors.

Figure 1 illustrates the NS exponential factor loadings with respect to five different values of λ from 0.01 to 0.09. It is clear that the bigger λ is, the faster will be the decay rate of the slope factor. The λ will also affect the shape of curvature factor. The loading on curvature factor is maximized at a maturity of 180, 62, 37, 27, and 21 months as λ is 0.01, 0.03, 0.05, 0.07, and 0.09, respectively.

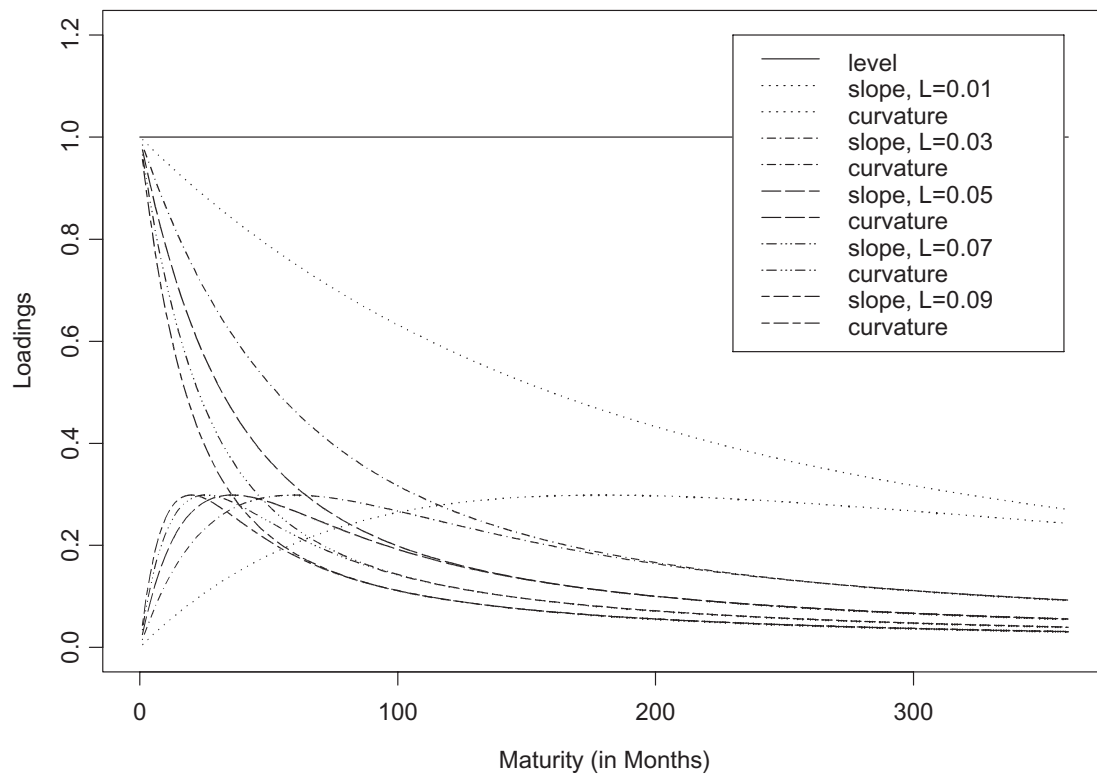


Figure 1. Factor loadings of Nelson–Siegel model by λ . Nelson–Siegel model is as follows:

$$y(\tau) = \beta_1 + \beta_2 \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} \right) + \beta_3 \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right)$$

where τ is the maturity of bond, which is from 0 to 360 months continuously. The parameter λ determines the rate of exponential decay

² Some authors define yield curve slope as $y_t(\infty) - y_t(0)$, which equals $\beta_{1t} - (\beta_{1t} + \beta_{2t}) = -\beta_{2t}$.

Despite its simplicity, the NS three-factor model is consistent with a variety of stylized facts regarding the yield curve. For example, the combination of three factors can capture different kinds of yield curves, including upward sloping, downward sloping, humped and inverted humped. Figure 2 presents the yield curve predicted by the NS model and the realized spot interest rates for AAA, AA, A+, A, BB+, and BB− corporate bonds in February 2004. The solid line is the predicted yield by NS model and the dot is the true value. The λ is chosen based on the best in-sample predictions. It seems that the range of λ between 0.06 and 0.08 is appropriate to fit the yield curve. Therefore, DL fix λ as 0.061 and DRA and YZ set λ as 0.077.

Dynamic Nelson–Siegel AR(1) models

Notwithstanding its success in cross-sectional interpolation, the NS model is not built for out-of-sample forecasts. DL extend it to the dynamic form. They propose a benchmark model: AR(1) method based on equation (1). First, they estimate three factors β_{1t} , β_{2t} , and β_{3t} by the least-squared method, period by period. Second, they use the estimated factors $\hat{\beta}_{1t}$, $\hat{\beta}_{2t}$, and $\hat{\beta}_{3t}$ to forecast the factors by a univariate AR(1) model:

$$\hat{\beta}_{j,t+h|t} = \hat{c}_j + \hat{\gamma}_j \hat{\beta}_{jt}, \quad j = 1, 2, 3 \quad (2)$$

Third, they convert the projected factors, which were computed in equation (2), to the predicted term structure of yields by combining the corresponding factor loadings as follows:

$$y_{t+h|t}(\tau_i) = \beta_{1,t+h|t} + \beta_{2,t+h|t} \left(\frac{1 - e^{-\lambda \tau_i}}{\lambda \tau_i} \right) + \beta_{3,t+h|t} \left(\frac{1 - e^{-\lambda \tau_i}}{\lambda \tau_i} - e^{-\lambda \tau_i} \right) \quad (3)$$

DL provides out-of-sample forecasts evaluations from the dynamic NS model as well as other forecast competitors. They include (1) random walk (the forecast is always no change), (2) slope regression, (3) Fama–Bliss forward rate regression, (4) Cochrane–Piazzesi (2005) forward curve regression, (5) AR(1) on yield levels, (6) vector AR(1): VAR(1) on yield levels, (7) VAR(1) on yield changes, (8) error correction model ECM(1) with one common trend, (9) ECM(1) with two common trends, and (10) ECM(1) with three common trends. The DL model concludes that other competing models are suboptimal compared to the AR(1) NS factor model for Treasury bonds from 1985:1 to 2000:12. Using Treasury and corporate yields from 1994 to 2006, YZ suggest that the NS three-factor AR(1) model is still very robust in the out-of-sample forecasting accuracy.

PARAMETER SENSITIVITY

As mentioned above, DL, DRA and YZ do not consider the sensitivity effect of parameter λ on the model's out-of-sample forecast performance because they focus on a comparison over different models. Instead, in this paper we explore the sensitivity of λ on the NS three-factor AR(1) model, which has been shown to be the most competitive model.

Data

We use the same dataset as used in YZ: end-of-month zero-coupon bond rate for S&P rating AAA, AA, A+, A, A−, BB+, BB−, B, and B− corporate bonds from December 1994 to April 2006, taken from Bloomberg. We analyze the raw zero-coupon yield data directly, unlike DL, who use the

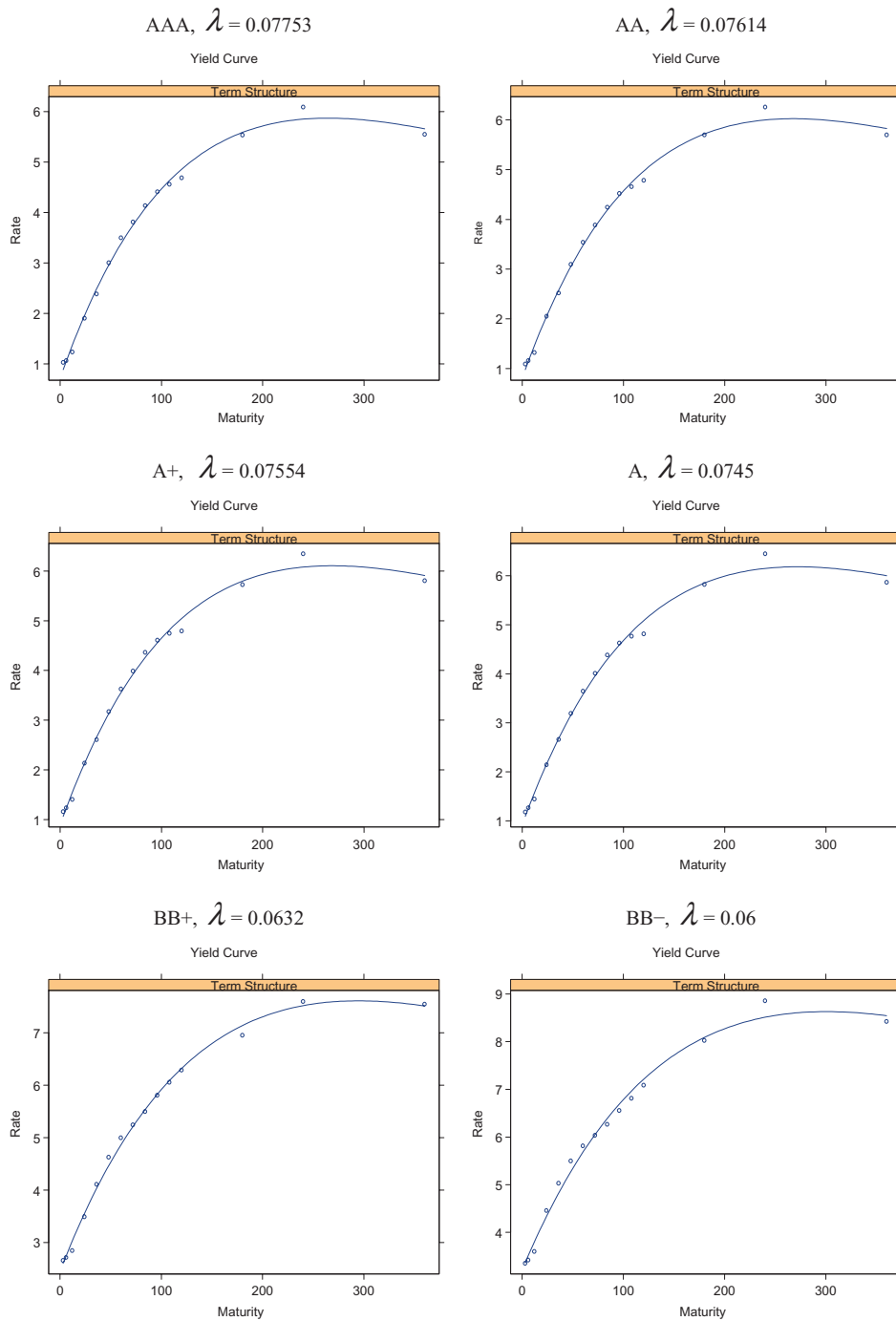


Figure 2. Nelson–Siegel corporate yield curve in-sample fit on February 2004

Fama–Bliss (1987) filter for cleaning their data. The maturities of bonds include 3, 6, 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 180, 240, and 360 months. We find two stylized facts about the sample mean. First, when the maturities become longer, interest rates are higher. This implies that, on average, the yield curves are upward sloping and term spreads are positive. The term-spread time series for each rating bond is shown in Figure 3. In each chart of Figure 3, we present the interest rate series of all maturities. When the bands between curves become wider, the term spread is larger. We notice that the term spread is narrow in 1999 and 2000 (pre-recession period), then becomes wider between 2002 and 2005 (recovery period), and shrinks in 2006.

Second, as the rating declines, the interest rate is higher. That said, on average, the credit spread is positive. Figure 4 shows the credit spread time series for each corporate bond. For example, the AAA credit spread is the difference between the AAA interest rate and Treasury yield at the same maturity, such as AAA 1-year interest rate minus Treasury 1-year interest rate, and the AAA 2-year interest rate minus Treasury 2-year interest rate. Figure 4 matches the stylized fact of the asymmetric impact in a recession. The recession in 2001 raised the credit spreads little in investment-grade bonds (above BBB–), while it increased the credit spreads much more in speculative-grade (below BBB–, also called non-investment-grade, high-yield) rated bonds.

Forecast evaluation

The success of a time-series model lies in its out-of-sample forecast performance. We assess the model's out-of-sample predictive accuracy on the 1, 6, 12, 36, and 60-month-ahead out-of-sample forecast. For instance, on the 1-month-ahead out-of-sample forecast, we use data from 1994:12 to 2004:4 as in sample and 2004:5 through 2006:4 (24 predictions) as out of sample. In order to estimate the parameters of the model based on the most up-to-date information available at the given time of forecast, the predictions are conducted via a rolling window. For example, we use data from 1994:12 to 2004:4 to forecast 2004:5; and we use data from 1995:1 to 2004:5 to forecast 2004:6. For a longer forecast horizon, in order to keep the out-of-sample size as 24 predictions, we have to decrease the in-sample size. For instance, for the 6-month-ahead forecast, we use data from 1994:12 to 2003:11; accordingly, our first prediction will be on 2004:5. There are two reasons for this. First, we will have a large enough out-of-sample size to test the model's forecast. Second, we can compare different models' predictabilities on smaller in-sample sizes, especially for long-term forecasts.

It is worth noting that we use iterated forecasts instead of direct forecasts for the multi-period-ahead predictions. Marcellino *et al.* (2005) argue that iterated forecasts are more efficient when the model is correctly specified and the forecast performance will improve with the prediction horizon. They conclude that the direct multi-period forecasts are more robust when the model is misspecified. The comparison of iterated and direct forecasts is not the main focus of the paper. Therefore, we use the former based on the assumption of correct model specification.

To evaluate the out-of-sample forecasting performance, we use root mean squared forecast errors (RMSE):

$$\left[\sum (y_{t+h} - \hat{y}_{t+h|t})^2 / 24 \right]^{1/2} \quad (4)$$

where y_{t+h} are the realized yields and $\hat{y}_{t+h|t}$ are the h -month-ahead predictions made by the AR(1) NS model. The smaller the RMSE, the better the model forecasts. For simplicity we only show the aggregate evaluation results on 1, 6, 12, 36, and 60-month-ahead forecasts of 3, 6, 12, 36, 60, 120,

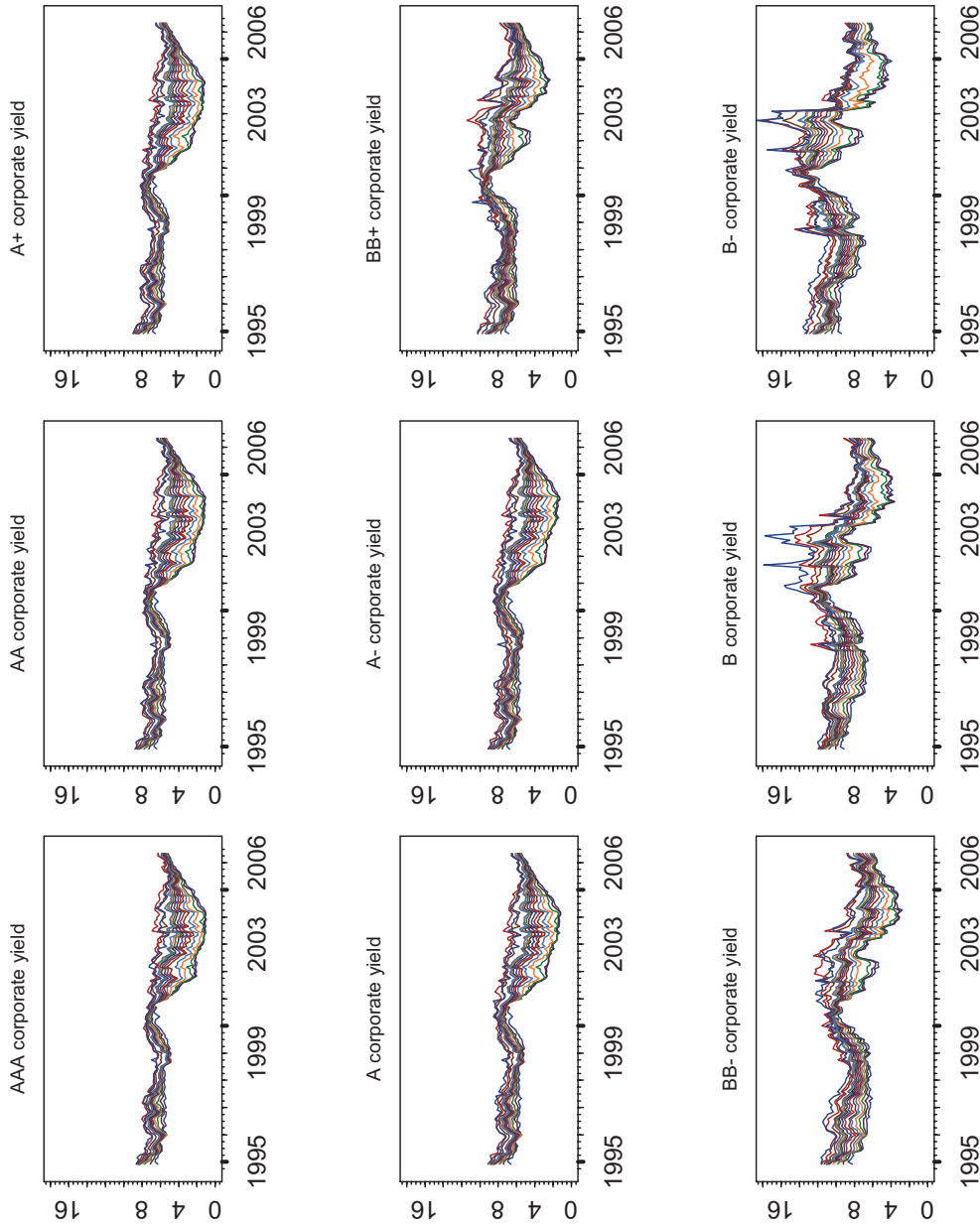


Figure 3. Interest yields time series and its implied term spread. In each chart we present all the maturities (3, 6, 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 180, 240, and 360 months) corporate interest rates series from 1994:12 to 2006:4

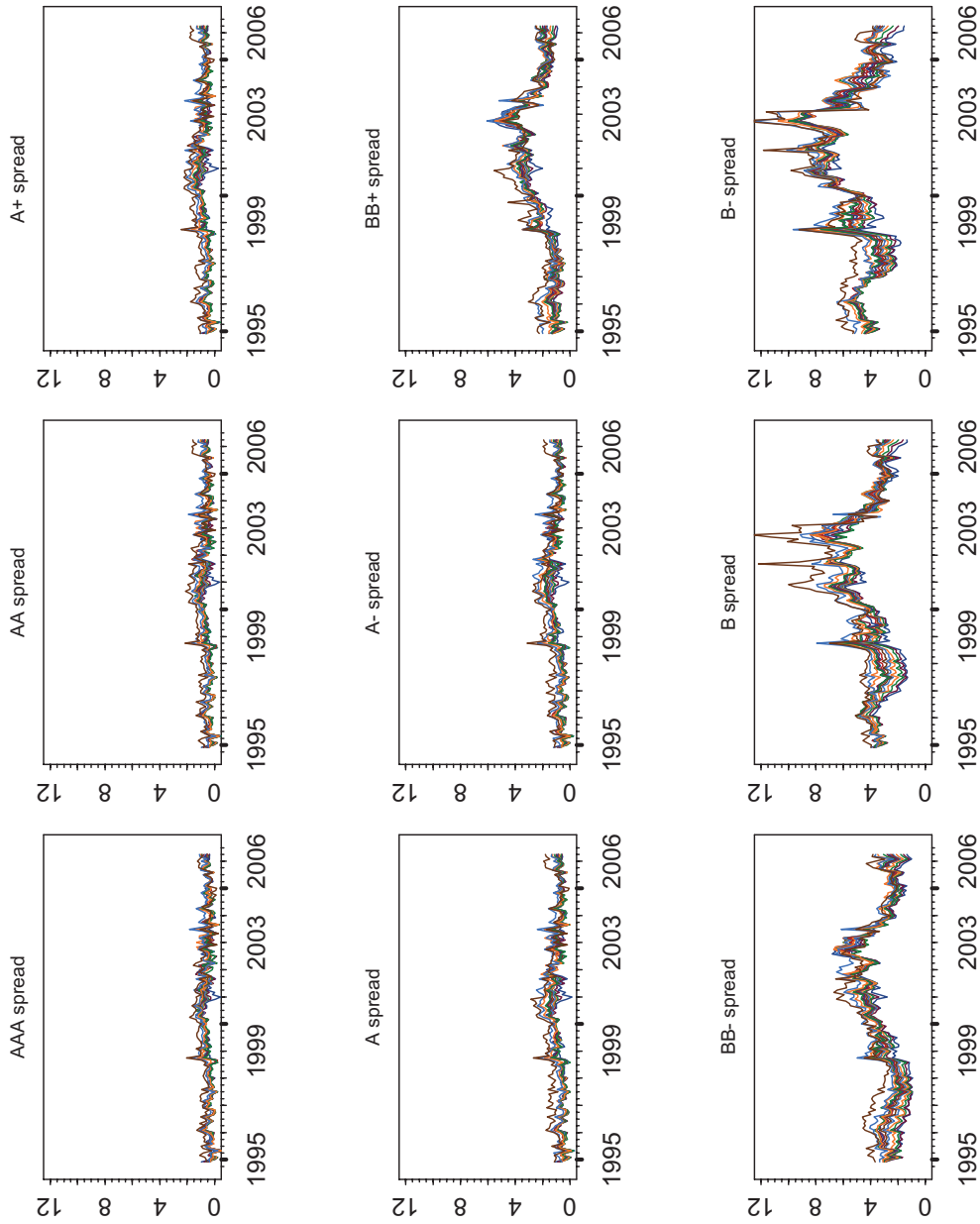


Figure 4. Credit spread time series. In each chart we present the maturities (12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 180, 240, and 360 months) corporate credit spreads, which is its corresponding interest rate minus Treasury interest rate series from 1994:12 to 2006:4

Table I. Root mean square errors of corporate yields 1994:12–2004:4. Out-of-sample forecasting by dynamic Nelson–Siegel AR(1) model with different λ

| λ | AAA | AA | A+ | A | A– | BB+ | BB– | B | B– |
|-----------|-------|-------|-------|-------|-------|-------|-------|--------|--------|
| 0.01 | 58.46 | 62.22 | 64.49 | 66.34 | 68.62 | 77.5 | 88.31 | 437.5 | 109.23 |
| 0.015 | 49.19 | 49.98 | 50.63 | 51.86 | 52.62 | 63.34 | 76.15 | 200.41 | 97.97 |
| 0.02 | 47.97 | 54.23 | 48.63 | 47.65 | 46.06 | 52.13 | 66.42 | 116.15 | 88.58 |
| 0.025 | 48.21 | 61.49 | 51.87 | 50.46 | 49.19 | 45.36 | 59.71 | 86.08 | 81.05 |
| 0.03 | 50.45 | 62.5 | 54.24 | 53.27 | 57.56 | 41.77 | 55.12 | 72.78 | 75.06 |
| 0.035 | 52.95 | 60.27 | 54.44 | 53.23 | 63.57 | 40.52 | 52.08 | 65.67 | 70.36 |
| 0.04 | 54.66 | 57.78 | 53.83 | 51.88 | 66.24 | 40.7 | 50.11 | 61.38 | 66.78 |
| 0.045 | 55.3 | 55.95 | 53.3 | 50.56 | 66.72 | 41.47 | 48.86 | 58.65 | 64.14 |
| 0.05 | 54.94 | 54.59 | 53 | 49.69 | 66.18 | 42.4 | 48.12 | 56.88 | 62.28 |
| 0.055 | 53.82 | 53.32 | 52.68 | 49.12 | 65.16 | 43.27 | 47.73 | 55.75 | 61.05 |
| 0.06 | 52.22 | 51.93 | 52.14 | 48.56 | 63.75 | 44.03 | 47.59 | 55.06 | 60.29 |
| 0.065 | 50.43 | 50.42 | 51.3 | 47.86 | 61.96 | 44.68 | 47.65 | 54.7 | 59.92 |
| 0.07 | 48.7 | 48.89 | 50.24 | 47.02 | 59.84 | 45.28 | 47.85 | 54.56 | 59.82 |
| 0.075 | 47.23 | 47.47 | 49.05 | 46.11 | 57.5 | 45.85 | 48.16 | 54.6 | 59.94 |
| 0.08 | 46.11 | 46.29 | 47.88 | 45.24 | 55.09 | 46.46 | 48.56 | 54.76 | 60.21 |
| 0.085 | 45.4 | 45.41 | 46.85 | 44.5 | 52.78 | 47.12 | 49.03 | 55.01 | 60.6 |
| 0.09 | 45.08 | 44.87 | 46.05 | 43.97 | 50.68 | 47.84 | 49.57 | 55.32 | 61.07 |
| 0.095 | 45.11 | 44.67 | 45.54 | 43.69 | 48.92 | 48.62 | 50.15 | 55.69 | 61.61 |
| 0.1 | 45.43 | 44.79 | 45.33 | 43.68 | 47.56 | 49.46 | 50.78 | 56.1 | 62.2 |
| 0.105 | 45.96 | 45.17 | 45.4 | 43.92 | 46.62 | 50.33 | 51.44 | 56.54 | 62.82 |
| 0.11 | 46.64 | 45.76 | 45.72 | 44.37 | 46.11 | 51.22 | 52.13 | 57 | 63.46 |
| 0.115 | 47.42 | 46.5 | 46.25 | 44.97 | 45.99 | 52.12 | 52.84 | 57.48 | 64.12 |
| 0.12 | 48.25 | 47.35 | 46.92 | 45.7 | 46.17 | 53.01 | 53.57 | 57.98 | 64.78 |
| 0.125 | 49.11 | 48.28 | 47.71 | 46.52 | 46.6 | 53.89 | 54.32 | 58.49 | 65.43 |

Nelson–Siegel model is as follows:

$$y_i(\tau_i) = \beta_{1i} + \beta_{2i} \left(\frac{1 - e^{-\lambda\tau_i}}{\lambda\tau_i} \right) + \beta_{3i} \left(\frac{1 - e^{-\lambda\tau_i}}{\lambda\tau_i} - e^{-\lambda\tau_i} \right)$$

where τ_i is the maturity of bond i , which is 3, 6, 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 180, 240, 360 months, respectively. The parameter λ determines the rate of exponential decay.

The NS AR(1) model is as follows:

$$\hat{\beta}_{j,t+h|t} = \hat{c}_j + \hat{\gamma}_j \hat{\beta}_{jt}, \quad j = 1, 2, 3$$

Root mean square errors (RMSEs) were calculated by the average of RMSEs of 1-month, 6-month, 12-month, 36-month, and 60-month-ahead out-of-sample forecasting on 3-month, 6-month, 12-month, 36-month, 60-month, 120-month, and 360-month corporate yields.

In-sample period is from December 1994 to April 2004. Out-of-sample period is from May 2004 to April 2006.

and 360-month maturities of corporate bonds in Table I. For example, each cell in Table I represents the sum of RMSE: 1 month ahead + 6 months ahead + 12 months ahead + 36 months ahead + 60 months ahead of the sum of 3, 6, 12, 36, 60, 120, and 360-month maturities RMSE given different λ values.

Out-of-sample forecast results

Surprisingly, very few of the lowest RMSEs were located in the range of 0.06–0.08 as chosen in DL, DRA and YZ. For investment-grade bonds the optimal choice of λ is 0.09, 0.095, 0.1, 0.1, and 0.115 for AAA, AA, A+, A, and A–, respectively. Their aggregate RMSEs are close: in the range

of 43.68–45.99. For speculative-grade bonds, the optimal choice of λ is 0.035, 0.06, 0.07, and 0.07 for BB+, BB-, B, and B-, respectively. Their aggregate RMSEs are diverse: from 40.52 to 59.82. If one has to choose a single value of λ to forecast all ratings of corporate bonds, the optimal choice would be 0.09, which produces the lowest sum of aggregate RMSE.

The findings suggest three things. First, the result of the model's prediction is sensitive to the value of λ . Second, the best in-sample fit does not guarantee the best out-of-sample forecasts, if we compare the results of Figure 2 with those of Table I. Nelson and Siegel (1987) argue a similar idea when they state: 'A more highly parameterized model that could follow all the wiggles in the data is less likely to predict well, in our view, than a more parsimonious model that assumes more smoothness in the underlying relation than one observes in the data.' Third, there is a big gap in the optimal value of λ between investment-grade and speculative-grade corporate bonds. How do we interpret the differentials in Table I? What are the underlying natures of factors on investment and speculative grade bonds? We offer an explanation by using the principal-component method in the next section.

PRINCIPAL-COMPONENT METHOD

Introduction to principal-component method

The principal-component method is a dimension reduction technique that transforms a number of correlated variables into a smaller number of uncorrelated variables from explaining the majority of the information in the sample covariance matrix of returns. Consider the factor representation for multiple time series data X_{it} , ($i = 1, \dots, N$, $t = 1, \dots, T$):

$$X_{it} = \Lambda F_t + e_{it} \quad (5)$$

where Λ is the factor loadings, F is the factor process, and e is the idiosyncratic disturbance. The factor loadings, factor process, and idiosyncratic errors are not observable. It is assumed that the disturbances are i.i.d., normally distributed and independent of the factor process. Normalizing the covariance matrix of F to be an identity matrix, the factor model covariance matrix is then

$$\Sigma = \Lambda \Lambda' + \Omega \quad (6)$$

where Ω is the diagonal covariance matrix of e_{it} . A root- T consistent and asymptotically normal estimator, $\hat{\Sigma} = (1/T) \sum_{t=1}^T (X_t - \bar{X})(X_t - \bar{X})'$, can be obtained.

Unobserved common factors

Here X_{it} is the first difference of interest rates to demean the data, i denotes each maturity of each corporate bond, and t is from 1994:12 through 2006:4. We order it from AAA 3-month, 6-month, 12-month, . . . , 360-month, AA 3-month, 6-month, . . . , to B- 360-month. Figure 5 shows the R^2 of the regressions for each maturity of all corporate bond time series against the first five factors by the principal-component method. These R^2 are plotted as bar charts with one chart for each factor and could be interpreted as factor loadings.

From Figure 5, it is apparent that factor loading patterns differ considerably between investment and speculative grade bonds. Loosely speaking, we can interpret that Factor 1 is the level factor affecting all corporate bonds. Factor 2 is the level factor affecting only investment-grade bonds.

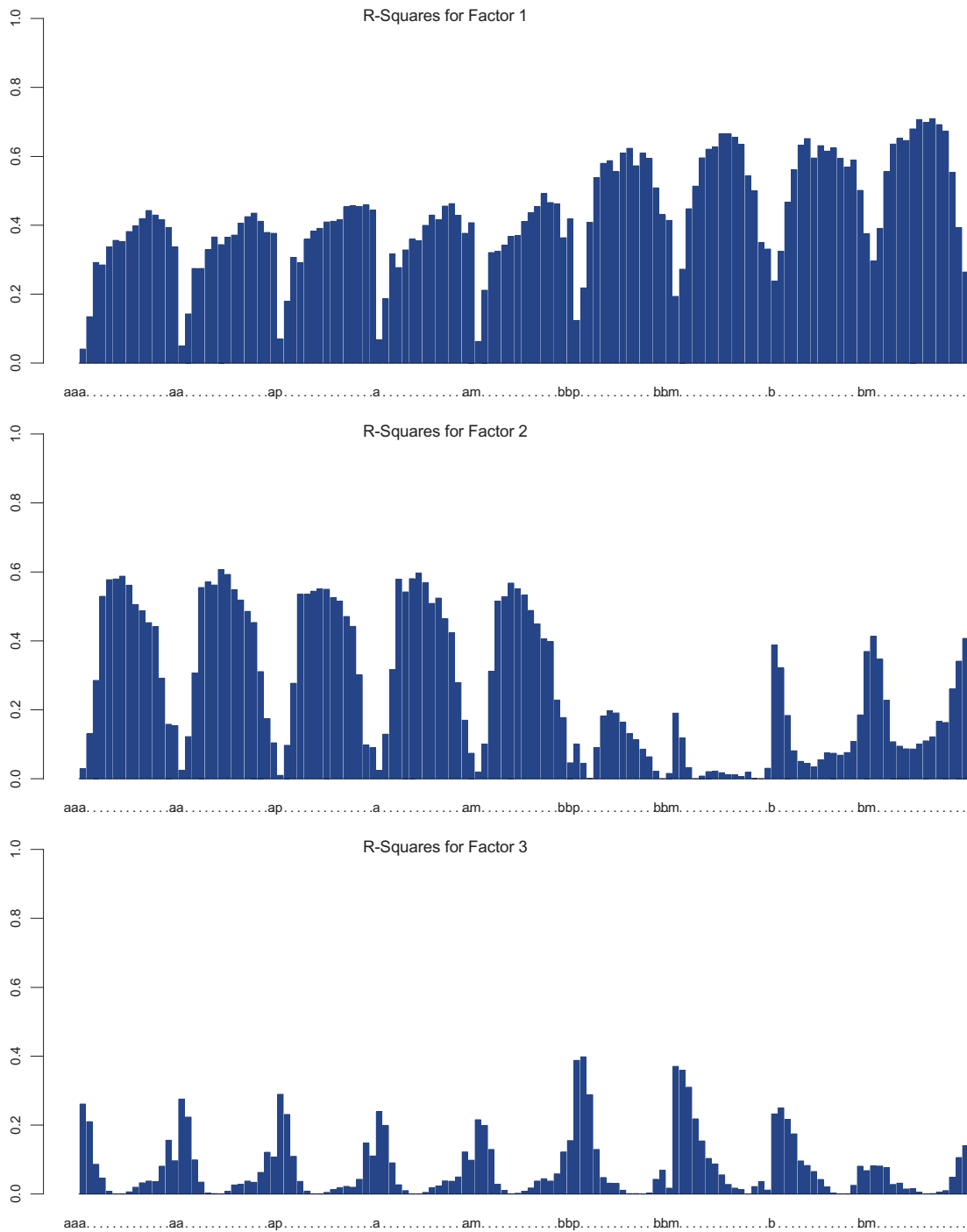


Figure 5. Factor loadings of all corporate yield changes

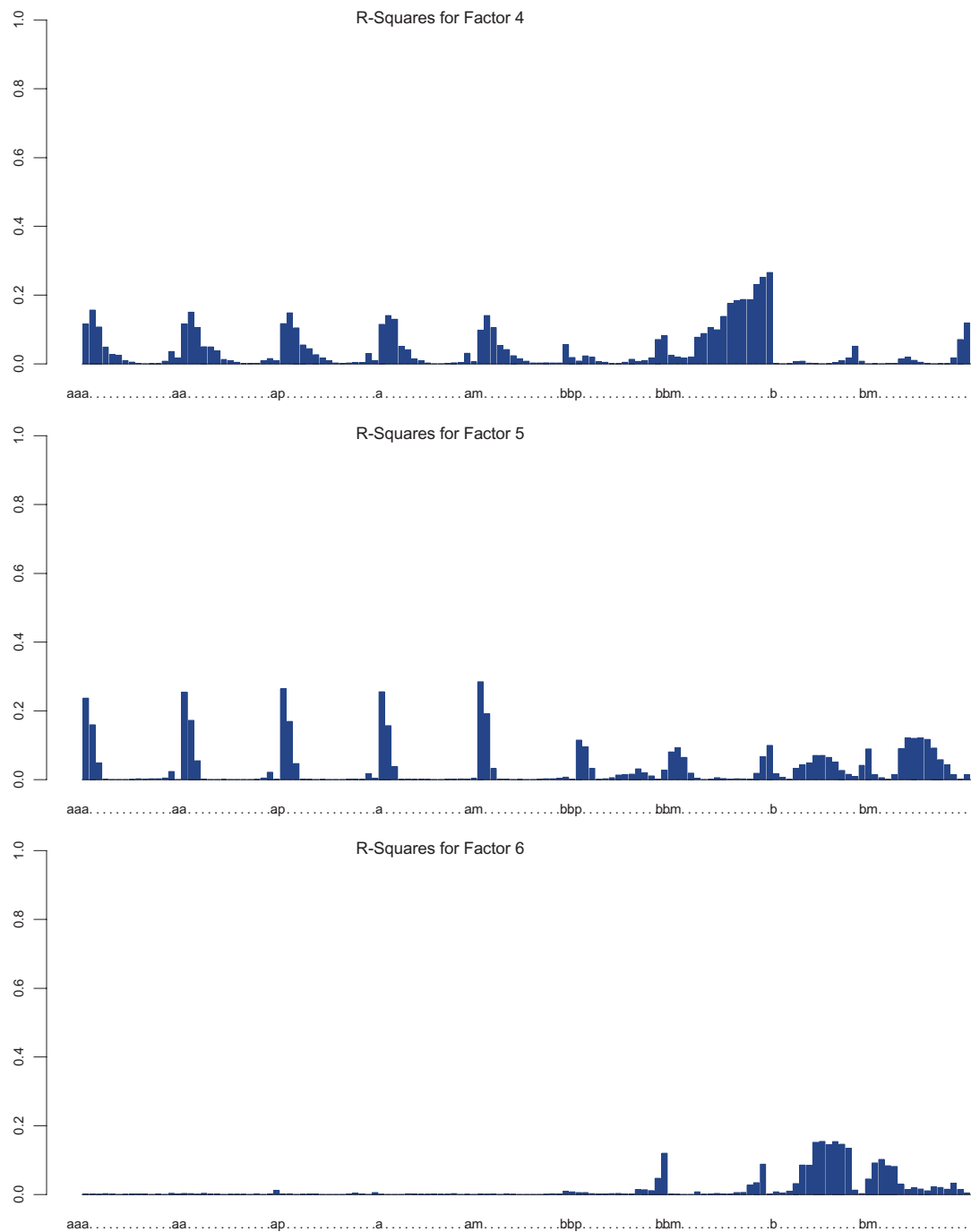


Figure 5. *Continued*

Factor 3 is the slope factor that loads heavily in the short ends of yields. Factor 4 is the curvature factor that loads the medium term of maturities of investment-grade bonds. Factors 5 and 6 are likely to be curvature factors for B and B- bonds.

To take a closer look at Factor 3 (slope factor), the decay rate of loadings for investment-grade bonds is faster than that for speculative-grade bonds. This finding explains the reason why the optimal values of investment-grade bonds are generally larger than those of speculative-grade bonds, as shown in Table I. According to Figure 1, we know that as λ rises there is a faster decay rate in the slope factor. Therefore, it is reasonable to see the dichotomy of the optimal value of λ because the underlying factor loadings indeed behave differently.

Figure 6 presents the factor loadings on 15 maturities of corporate bonds at the individual rating level (AAA, BB+). For AAA corporate yields, Factor 1 is the level factor, Factor 2 is the slope factor, and Factor 3 is the curvature factor. For BB+ corporate yields, Factor 1 is the level factor, Factor 2 is the curvature factor, and Factor 3 is the slope factor. It is reasonable to get higher RMSE from speculative-grade bonds than that from investment-grade bonds because the former is more volatile than the latter. For BB+ bonds, however, we get a lower RMSE (40.52) even compared to investment-grade bonds (Table I). The possible reason might be found in the curvature factor in Figure 6. We can see that the loading of the curvature factor (from 3 months to 96 months) on the BB+ bond mimics the curvature factor in the NS model well, so it is not surprising to see the lowest RMSE.

Number of factors

Is a three-factor model sufficient to explain the major variation of corporate yield changes? Will there be any gain in forecasting accuracy by adding one or two factors to the model in order to capture the dynamics of default or credit risk? Litterman and Scheinkman (1991) argue that three factors are sufficient to explain the movements of Treasury yields. To find the optimal number of factors on corporate bonds, we implement the principal-component method on each rating corporate bond to get the marginal R^2 (variation explained) for the first five factors. Table II displays the results. For all corporate bonds Factor 1 (level factor) explains most of the variation of yields, ranging from 66.5% to 78.1%. Factor 2 explains the variation from 10.6% to 14%. Factor 3 explains the variation from 3.7% to 12.6%. The first three factors explain around 92% of the yield variations. Surprisingly, there is no evidence to support the fourth factor, even for speculative-grade bonds.

Svensson (1994) proposes an extended NS model by adding one factor with additional λ as follows:

$$y_t = \beta_{1t} + \beta_{2t} \left(\frac{1 - e^{-\lambda_1 \tau}}{\lambda_1 \tau} \right) + \beta_{3t} \left(\frac{1 - e^{-\lambda_1 \tau}}{\lambda_1 \tau} - e^{-\lambda_1 \tau} \right) + \beta_{4t} \left(\frac{1 - e^{-\lambda_2 \tau}}{\lambda_2 \tau} - e^{-\lambda_2 \tau} \right) \quad (7)$$

He argues that this four-factor model provides a better in-sample fit, especially for a richer structure of yields, and therefore has better estimations of forward rates. We apply his model in an AR(1) dynamic form and conduct the out-of-sample forecast evaluations with a range of λ_1 and λ_2 . We find that Svensson's model produces inferior results compared to the NS three-factor model.

CONCLUSION

In this paper, we investigate the sensitivity of out-of-sample forecasting performance over a span of different parameters of λ in the dynamic Nelson–Siegel three-factor AR(1) model. Using the S&P

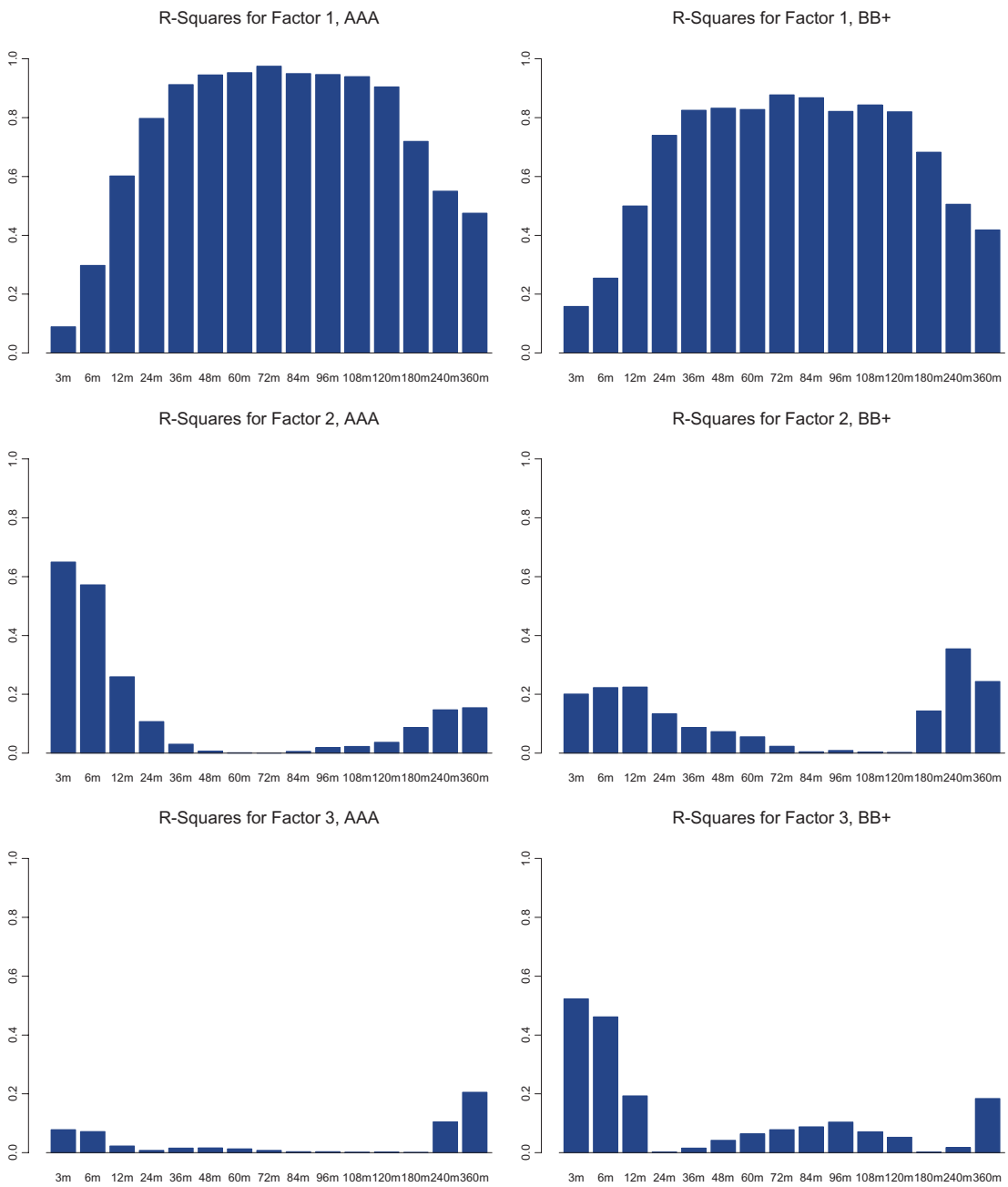


Figure 6. Factor loadings of AAA and BB+ corporate yield change

Table II. Relative importance of factors (R^2) (percentage)

| | Factor 1 | Factor 2 | Factor 3 | First three factors explained | Factor 4 | Factor 5 |
|-----|----------|----------|----------|-------------------------------|----------|----------|
| AAA | 0.737 | 0.14 | 0.037 | 0.914 | 0.032 | 0.021 |
| AA | 0.739 | 0.134 | 0.049 | 0.922 | 0.026 | 0.02 |
| A+ | 0.744 | 0.13 | 0.057 | 0.931 | 0.021 | 0.017 |
| A | 0.743 | 0.12 | 0.061 | 0.925 | 0.024 | 0.016 |
| A- | 0.732 | 0.123 | 0.063 | 0.919 | 0.026 | 0.018 |
| BB+ | 0.665 | 0.119 | 0.126 | 0.91 | 0.03 | 0.026 |
| BB- | 0.733 | 0.12 | 0.075 | 0.927 | 0.025 | 0.018 |
| B | 0.721 | 0.122 | 0.082 | 0.925 | 0.024 | 0.024 |
| B- | 0.781 | 0.106 | 0.058 | 0.945 | 0.021 | 0.012 |

Data are from 1994:12 to 2006:4. Factors are estimated by principal-component method.

rating AAA, AA, A+, A, A-, BB+, BB-, B, and B- corporate bonds from 1994:12 to 2006:4, we find that the ad hoc selection of λ is not optimal. Moreover, we find considerable differences of factor loadings between investment-grade and speculative-grade corporate bonds from 1994:12 to 2006:4. Based on the principal-component method, we suggest that the three factor-model is sufficient to explain the main variations of corporate yields: about 92%. The parsimonious Nelson–Siegel three factor AR(1) model is still competitive in the out-of-sample forecasting of corporate yields.

The findings also imply two things. First, good in-sample fits do not guarantee good out-of-sample forecasts. It is more important to catch the underlying relations of data than to fit superficially observed data. Second, there is no need to explore the fourth factor for default risk and credit spreads of corporate bonds because their dynamics have been captured by three factors, especially the level factor.

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Authors' biographies:

Wei-Choun Yu is an Assistant Professor in the Economics and Finance Department at Winona State University. He also serves on the board of directors at the Minnesota Economic Association. He received his PhD degree in economics at the University of Washington in 2006. Yu's research interests are in time series econometrics, finance and macroeconomics, in particular linear, nonlinear, and volatility modeling and forecasting. He has various publications in the fields of econometrics and finance.

Donald M. Salyards is a Professor of Economics at Winona State University. He received his PhD degree in economics at Kansas State University in 1975. Salyards focuses most of his time establishing and funding new firms as an angel investor. Comfortex, Inc. and Winona Pattern & Mold Company are his most successful entrepreneurial endeavors. Salyards runs the entrepreneurship program at Winona State University.

Authors' address:

Wei-Choun Yu and **Donald Salyards**, Economics and Finance Department, Winona State University, Somsen 319E, Winona State University, Winona, MN 55987, USA.