

Dynamic Hedging Performance with the Evaluation of Multivariate GARCH Models: Evidence from KOSTAR Index Futures

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Abstract

The paper examines the hedging performance of the conventional OLS model and a variety of dynamic hedging models for the in-sample and out-of-sample periods of Korean daily KOSDAQ STAR (KOSTAR) index futures. We employ the rolling OLS and various popular multivariate GARCH models to estimate and forecast the conditional covariances and variances of KOSTAR spot and futures returns. The paper finds that dynamic hedging methods outperform the conventional method for the out-of-sample period. In particular, the simple rolling OLS is superior to all the other popular multivariate GARCH models.

Keywords: KOSTAR index futures; Hedge performance; Rolling OLS; Multivariate GARCH; Utility function

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1. Introduction

Hedging is to reduce the risk and offset the loss generating from the spot position of commodities, foreign exchange, and financial assets with the use of derivatives.

Previous researches, such as Working (1953), Johnson (1960), Stein (1961), Ederington (1979), and Figlewski (1984), have investigated the conventional optimal hedging models, which restrict the hedge ratio to be constant over time. Recent studies, such as Baillie and Myers (1991), Myers (1991), Kroner and Sultan (1993), Ghosh (1993), Park and Switzer (1995), and Lee et al. (2006) present the improvement of hedging performance by time-varying minimum-variance hedge ratios based on either conditional second moments or time-varying coefficient of the model.

On the other hand, following the seminal work of multivariate generalized autoregressive conditional heteroskedasticity models (GARCH) by Bollerslev, Engle and Wooldridge (1988), a series of multivariate GARCH model extensions have been proposed. It is well known that GARCH models could well describe the in-sample distribution of stock and futures returns. However, few of these multivariate GARCH variants have been evaluated based on their out-of-sample forecasting performances. This paper bridges the gap of the application and evaluation of various GARCH models in the in-sample and out-of-sample dynamic hedging. In particular, we estimate and

forecast the variances and covariances of Korean stock and futures based on several popular bivariate GARCH models. Furthermore, we evaluate the performance of variance reduction for the hedged portfolio according to these models' predictions. Finally, we analyze the mean variance utility for the out-of-sample period to consider the transaction costs induced by the daily portfolio rebalance.

The rest of this paper is organized as follows. Section 2 illustrates the models that we employ. Section 3 explains the data and shows the summary statistics. Section 4 presents the model estimation and evaluation of hedging performance and investor's utility. Section 5 concludes the paper.

2. Models and Methodology

This study employs seven models. The first one is the conventional ordinary least square (OLS),

$$r_{s,t} = \alpha + \beta r_{f,t} + e_t \quad (1)$$

where $r_{s,t}$ is the stock spot return and $r_{f,t}$ is the stock futures return. The OLS estimator

$$\beta^* = \frac{\sigma_{sf}}{\sigma_f^2} \quad (2)$$

where β^* is the optimal hedge ratio which will maximize utility function of an investor

who faces the mean-variance expected utility function. This conventional hedging strategy assumes that the investor holds one unit in long position in the spot stock market. To maximize his utility as well as minimize the variance of his long position, he holds the β^* unit of short position in the futures market. When β is one, it is called naïve hedge strategy.

Despite its simplicity and zero transaction cost, the conventional hedge model cannot incorporate and update the information from data, which could be a crucial problem when the out-of-sample is very volatile. Therefore, we propose a variant of OLS model, which is the rolling-window OLS model. We estimate OLS estimator β_t^* based on the in-sample data. When we estimate the out-of-sample hedge ratio, we use the latest available data instead of fixed in-sample data. For example, we use in-sample $r_1 \square r_t$ to get OLS estimator β_t^* as the optimal hedge ratio for the first one-step-ahead out-of-sample data r_{t+1} ; then we use $r_2 \square r_{t+1}$ to get OLS estimator β_{t+1}^* as the optimal hedge ratio for the second one-step-ahead out-of-sample data r_{t+2} . In other words, the in-sample OLS estimation is rolling over as we hedge in the out-of-sample. This method could allow us to incorporate the latest information and discard the out-dated information as we go through the out-of-sample as well as the real-time hedging.

Since the joint distribution of stock spots and futures market could be time-varying, we also consider the alternative models from the multivariate GARCH family.

The third model is the simplified diagonal VECH GARCH (1,1) (DVEC GARCH)

model, introduced by Bollerslev et al. (1988),

$$\begin{aligned} r_{s,t} &= \alpha_s + e_{s,t} \\ r_{f,t} &= \alpha_f + e_{f,t} \end{aligned} \quad (3)$$

$$\begin{bmatrix} e_{s,t} \\ e_{f,t} \end{bmatrix} \Bigg| \psi_{t-1} \square N(0, H_t) \quad (4)$$

$$H_t = U + A \otimes e_{t-1} e_{t-1}' + B \otimes H_{t-1} \quad (5)$$

$$\begin{aligned} H_t &= \begin{bmatrix} h_{ss,t} & 0 \\ h_{fs,t} & h_{ff,t} \end{bmatrix} = \begin{bmatrix} u_{ss} & 0 \\ u_{fs} & u_{ff} \end{bmatrix} + \begin{bmatrix} a_{ss} & 0 \\ a_{fs} & a_{ff} \end{bmatrix} \begin{bmatrix} e_{s,t-1} e_{s,t-1} & 0 \\ e_{f,t-1} e_{s,t-1} & e_{f,t-1} e_{f,t-1} \end{bmatrix} \\ &+ \begin{bmatrix} b_{ss} & 0 \\ b_{fs} & b_{ff} \end{bmatrix} \begin{bmatrix} h_{ss,t-1} & 0 \\ h_{fs,t-1} & h_{ff,t-1} \end{bmatrix} \end{aligned} \quad (6)$$

where Equation (3) is the mean equation of the model; e_t is the innovation term, which follows a normal distribution with mean zero and conditional variance H_t ; ψ_t is the information set at time $t-1$, and \otimes is the Hadamard product. Equation (4) and (5) show that conditional variance follows an ARMA (1,1) process, which depends on its last-period variance and last-period squared residual. As shown in Equation (6), we only consider the lower triangular part of the symmetric metrics of U , A , and B . The covariance matrix must be positive semi-definite (PSD) but H_t in the DEVC model

cannot be guaranteed to be PSD. Therefore, we consider the fourth model – Matrix Diagonal GARCH (1,1) model, modified from Bollerslev et al. (1994),

$$H_t = UU' + AA' \otimes e_{t-1}e_{t-1}' + b \otimes H_{t-1} \quad (7)$$

where b is just a scalar. Equation (7) is a simple PSD version of the DEVC model.

Although the Matrix Diagonal model has PSD covariance matrices, the dynamics in the covariance matrices is still restricted. Engel and Kroner (1995) propose the famous BEKK (Baba-Engle-Kraft-Kroner) GARCH (1,1) model, which would be our fifth model,

$$H_t = UU' + A(e_{t-1}e_{t-1}')A' + BH_{t-1}B' \quad (8)$$

where Equation (8) not only guarantees the PSD but also allows unrestricted matrices,

where variances of the two variables have concurrent impact on each other by

estimating two more parameters a_{sf} and b_{sf} . The sixth model is the constant

conditional correlation - CCC GARCH (1,1) model, suggested by Bollerslev (1990),

which assumes a time-invariant correlation, ρ , as shown in Equation (9).

$$H_t = \begin{bmatrix} h_{ss,t} & h_{sf,t} \\ h_{sf,t} & h_{ff,t} \end{bmatrix} = \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix} \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} \begin{bmatrix} h_{s,t} & 0 \\ 0 & h_{f,t} \end{bmatrix} \quad (9)$$

The last model is the principal component GARCH (1,1) model,

$$H_t = \Lambda_t \Delta_t \Lambda_t' \quad (10)$$

where Λ is an orthogonal matrix such that $\Lambda \Lambda' = I$ is an identity matrix; Δ is a

diagonal matrix, of which the diagonal elements are the eigenvalues of H and the columns of Λ denote the eigenvectors of H .

It is worth noting that we do not incorporate the error correction terms into the mean equations of all the above models. It is because we fail to reject the null hypothesis of no cointegration for the KOSTAR spot and futures returns, based on the two step residual-based test from Engel and Granger (1988).

3. Data

The spot and futures data used in this paper are from Korea Securities Dealers Automated Quotation (KOSDAQ) market. The KOSDAQ market was established in July 1996 to provide investors with high risk and high return investment opportunities in small/ middle size or high-tech firms, like NASDAQ in the U.S. Because the Korean government relaxed regulation and provided tax incentives on KOSDAQ market, the total number of listed firms on KOSDAQ has been growing rapidly. As of October, 2007, the total number of listed companies on KOSDAQ is 1,009, which is larger than that on the Korea Stock Exchange (944). Its market value is 100 trillion won (\$107 billion) and the average trading volume per day is 600 million shares.

As the KOSDAQ market grows rapidly, the demand of market investors to

hedge the market risk in stock price has also increased. Therefore, Korea Exchange (KRX) introduced the KOSDAQ 50 index futures on January 30, 2001. It is composed of 50 companies with the highest market value in the KOSDAQ market. However, the KOSDAQ 50 index futures cannot satisfy investors who wanted to reduce the spot risk.⁴ To solve this problem, the KRX provided a new KOSDAQ index – the KOSDAQ STAR (KOSTAR) index. KOSTAR consists of 30 constituents selected from all listed stocks on KOSDAQ based on factors such as liquidity, financial requirements, and representability in the market. It is a value-weighted index and has been published since January 26, 2004.

The KOSTAR index futures was created based on of the underlying asset of KOSTAR on November 7, 2005. The trading volume of KOSTAR index futures in 2006 and 2007 were 90,147 and 22,177 contracts, respectively. The average trading volume per day in 2006 and 2007 were 366 and 90, respectively. In addition, the open interests of 2006 and 2007 were 44,625 and 181,287 contracts. These figures help us to assume that there is enough liquidity for investors to perform the hedging. Note that we use the price of the nearest futures contract among several outstanding contracts at any given time. To avoid the illiquidity and expiration effects, we roll over to the next nearest

⁴ The low trading volume of KOSDAQ 50 spot and futures markets results from the poor representative

contract one week before the expiration of the current contract. The details of the KOSTAR futures contracts are shown in Appendix.

Table 1 reports the summary statistics of the returns of the KOSTAR index spot and futures from November 8, 2005 through November 8, 2007. The returns are computed as the first difference in the logarithm of price multiplied by 100. To perform model estimation, forecasting, and evaluation, the whole data period is divided into two subsamples: in-sample (November 8, 2005 to May 31, 2007, 389 observations) and out-of sample (June 1, 2007 to November 8, 2007, 108 observations). The returns and standard deviations of out-of-sample are higher than those of in-sample. According to the skewness and kurtosis, both samples do not follow the normal distribution. In particular, we find higher kurtosis from spot return and futures return of out-of-sample.

4. Estimation Results and Hedging Performance

In this section, we present the estimations of the GARCH model family, its corresponding time-varying hedge ratios, and the hedge performance during both in-sample and out-of-sample periods.

of the KOSDAQ market.

4.1. Estimation Results

In-sample parameter estimates from various multivariate GARCH models are presented in Table 2. The parameters are estimated by S-PLUS 7. In Table 2, most of the parameters are significant except for the BEKK model. As preliminary model selection checks, the principal component GARCH model having the highest Akaike Information Criterion (AIC: 2176.17) and Bayesian Information Criterion (BIC: 2215.80) is the best in-sample fit model. On the contrary, the DVEC and the CCC GARCH models have the lowest AIC (2129.48) and BIC (2162.14), respectively.

Figures 1 to 5 show the dynamic hedge ratios from Equation (2), which uses time-varying variances and covariances of five multivariate GARCH models during the in-sample period. The horizontal line is the constant hedge ratio (0.82) based on the OLS estimation from Equation (1). The hedge ratios are time-varying as conditional covariances and variance are changing over time. By and large, the dynamic hedge ratios of DEVC, matrix-diagonal and BEKK have a similar pattern and are mean-reverting to the conventional constant hedge ratio. However, the CCC and principal component models have quite different dynamic patterns.

4.2. Hedging Performance

In order to evaluate the performances of different hedge models, we construct the hedged portfolios based on the optimal hedge ratios for each trading day and then we compute the variances of the hedged portfolios,

$$\text{Var}(r_{s,t} - \beta^* r_{f,t}) \quad (11)$$

where β^* is the optimal hedge ratio calculated from the models mentioned in Section 2.

We show the variance reduction, which is computed as the difference between the sample variance of the unhedged spot position and the estimated variance of the hedged portfolio of each model divided by the sample variance of the unhedged position. The hedge performances of models are divided into in-sample and out-of-sample periods.

Table 3 presents the results. During the in-sample period, the matrix-diagonal GARCH (variance: 0.4248 and variance reduction: 82.64%) outperforms other models. It is surprising to see that the OLS - conventional constant hedge model performs well and is only inferior to the matrix-diagonal model. During the out-of-sample period, the principal component GARCH model (variance: 0.2056 and variance reduction: 95.52%) is superior to other models. Without the updated information and dynamic variance structure, the OLS model performs poorly during out-of-sample period (variance: 0.2721 and variance reduction: 94.07). Interestingly, the simple rolling-window OLS

model (variance: 0.2088 and variance reduction: 95.45%) could easily beat various popular multivariate GARCH models, such as BEKK GARCH (variance: 0.2383 and variance reduction: 94.81%) and CCC GARCH (variance: 0.2250 and variance reduction: 95.10%). This result suggests that the time-varying hedge ratio from the updated information/data through rolling in-sample is more competitive than that from the dynamic structure of variance-covariance of models, such as in the GARCH family.

4.3. Utility Comparisons

From Table 3, we are not clear whether the dynamic hedge model is more desirable than the conventional one if we consider transaction costs from daily portfolio rebalances. To better understand the economic significance and practical usefulness of dynamic hedging strategies, we investigate the mean-variance utility for a representative investor,

$$EU(r_{s,t} - \beta^* r_{f,t}) = E(r_{s,t} - \beta^* r_{f,t}) - x - \gamma \text{Var}(r_{s,t} - \beta^* r_{f,t}) \quad (12)$$

where x is the transaction cost, which reduces the utility; and γ is the degree of risk aversion. We assume that the expected return to the hedged portfolio is zero and the value of γ is 4.⁵ Therefore, the utility from hedging is $-x - 4\text{Var}(r_{s,t} - \beta^* r_{f,t})$. The investor rebalances his portfolio daily during the out-of-sample period so the number of

⁵ The assumption of the value of γ (4) is based on the main findings in the literature (for instance,

transactions is 108. A typical round-trip transaction cost is around 0.00072% of a KOSTAR futures contract value. In our simulation, we consider the four cost units: 0.0007, 0.0008, 0.0009, and 0.001. The results are shown in Table 4.

When the transaction cost is 0.0007, for example, the utility from the conventional OLS model is $-1.088 (-4 \times 0.2721)$ and the utility from the rolling OLS is $-0.9106 (-0.0007 \times 108 - 4 \times 0.2088)$. From Table 4, it is evident and not surprising that any dynamic hedging model is superior to the conventional one.

5. Conclusion

This study presents dynamic hedging models to calculate risk-minimizing hedge ratios in the daily KOSTAR index futures from November 8, 2005 through November 8, 2007. To compare the hedging performance of conditional hedging models with that of a conventional hedging method, we employ the rolling OLS and various multivariate GARCH (1,1) models during both in-sample and out-of-sample periods. We suggest three findings. First, all dynamic hedging models outperform the conventional hedging model (OLS) in the out-of-sample period. Second, using the mean-variance utility function, dynamic hedging models remain desirable even though we consider

Grossman and Shiller (1981) and Kroner and Sultan (1993).

transaction cost induced by daily portfolio rebalances. Third, we find that the simple rolling model is superior to all the multivariate GARCH models during the out-of-sample period. This result suggests that the time-varying hedge ratio from the updated information/data through rolling in-sample is more competitive than that from the dynamic structure of variance-covariance of models, such as in the GARCH family.

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Table 1
Summary Statistics for Spot and Futures Returns of KOSTAR Index Contracts

	Whole Sample		In Sample		Out-of-sample	
	Spot	Futures	Spot	Futures	Spot	Futures
Observations	497	497	389	389	108	108
Mean	0.0960	0.0965	0.0753	0.0744	0.1705	0.1759
SD	1.7048	1.8259	1.5643	1.7155	2.1429	2.1854
Skewness	-0.9359	0.8405	0.9085	0.8202	-0.0955	0.8791
Kurtosis	5.9768	5.5603	3.4763	4.2641	7.1524	6.4724

Whole Sample period: Nov. 8, 2005- Nov. 8, 2007

In Sample period: Nov. 8, 2005- May 31, 2007

Out-of-sample period: June 1, 2007- Nov. 8, 2007

Table 2
Parameter Estimates of Models for the In-Sample Period

	DVEC-GARCH	Matrix-Diagonal GARCH	BEKK GARCH	CCC GARCH	Principal Component GARCH
α_s	0.2187 (3.296***)	0.2023 (3.050***)	0.1996 (2.877***)	0.2192 (3.319***)	0.2435 (2.406**)
α_f	0.2215 (3.115***)	0.1910 (2.638***)	0.2128 (2.837***)	0.2140 (3.005***)	0.0219 (1.226)
u_{ss}	0.1639 (6.918***)	0.3883 (7.275***)	0.4806 (5.549***)	0.2305 (3.494***)	0.3606 (2.157**)
u_{fs}	0.1321 (7.357***)	0.4186 (7.748***)	0.2347 (2.620***)	-	-
u_{ff}	0.1594 (6.943***)	0.1087 (2.869***)	0.0718 (0.643)	0.1862 (3.454***)	0.0168 (1.170)
a_{ss}	0.1232 (12.664***)	0.3388 (11.091***)	0.1587 (1.399)	0.1642 (6.007***)	0.1781 (3.860***)
a_{fs}	0.0969 (19.832***)	0.3012 (10.107***)	-0.1293 (-1.247)	-	-
a_{sf}	-	-	0.2552 (2.715***)	-	-
a_{ff}	0.1054 (10.654***)	0.1233 (10.182***)	0.3932 (4.429***)	0.1168 (5.661***)	0.1513 (2.726***)
b_{ss}	0.7928 (81.824***)	-	0.7699 (7.389***)	0.7358 (15.735***)	0.7521 (11.195***)
b_{fs}	0.8298 (578.337***)	-	0.0407 (0.483)	-	-
b_{sf}	-	-	0.0812 (1.103)	-	-
b_{ff}	0.8221 (94.585***)	-	0.9138 (16.700***)	0.8087 (21.782***)	0.6705 (6.236***)
AIC	AIC(11): 2129.476	AIC(9): 2143.062	AIC(13): 2154.856	AIC(8): 2130.434	AIC(10): 2176.165
BIC	BIC(11): 2173.075	BIC(9): 2178.735	BIC(13): 2206.382	BIC(8): 2162.143	BIC(10): 2215.801

The in-sample data period is from November 8, 2005 to May 31, 2007. DVECH-GARCH is diagonal VECH GARCH, CCC GARCH is the constant conditional correlation GARCH. ***, ** indicate significance at a 1% and 5% levels, respectively.

Table 3
Hedging Performance of Models

	Variance		Variance Reduction	
	In Sample	Out-of-sample	In Sample	Out-of-sample
Unhedged	2.4470	4.5920	-	-
OLS	0.4253	0.2721	82.62%	94.07%
Rolling OLS	0.4253	0.2088	82.62%	95.45%
DVEC-GARCH	0.4256	0.2105	82.61%	95.42%
Matrix-Diagonal GARCH	0.4248	0.2111	82.64%	95.40%
BEKK GARCH	0.4302	0.2383	82.42%	94.81%
CCC GARCH	0.4311	0.2250	82.38%	95.10%
Principal Component GARCH	0.4326	0.2056	82.32%	95.52%

The in-sample data period is from November 8, 2005 to May 31, 2007 and the out-of-sample data period is from June 1, 2007 to November 8, 2007. Variance stands for the variance of the hedged portfolio calculation based on Equation (11). The percentage variance reductions are calculated as the differences of the variance of the unhedged position and the estimated variances of alternative models over the variance of the unhedged position multiplied by 100.

Table 4
Utility Comparisons for the Out-of-Sample Period

	Transaction Cost			
	0.0007	0.0008	0.0009	0.001
OLS	-1.088	-1.088	-1.088	-1.088
Rolling OLS	-0.9106	-0.9214	-0.9322	-0.943
DVEC-GARCH	-0.9174	-0.9282	-0.9322	-0.9498
Matrix-Diagonal GARCH	-0.9291	-0.9309	-0.9417	-0.9525
BEKK GARCH	-1.0286	-1.0394	-1.0502	-1.0610
CCC GARCH	-0.9751	-0.9860	-0.9968	-1.0076
Principal Component GARCH	-0.8982	-0.9090	-0.9198	-0.9306

The out-of-sample data period is from June 1, 2007 to November 8, 2007. The table shows the utility of the portfolio assuming that an investor rebalances his portfolio 108 times except OLS model. The potential gains in utility from the reduced variance offset the transaction costs that must be incurred. The utility of this portfolio is calculated from Equation (12).

Figure 1. Dynamic Hedge Ratio - Diagonal VEC GARCH Model

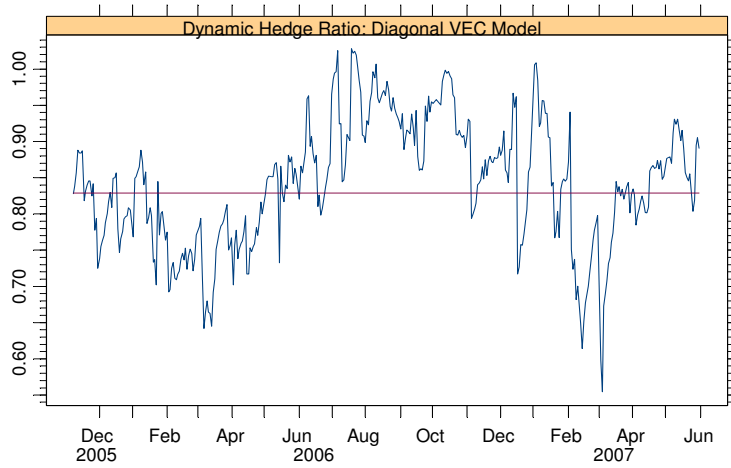
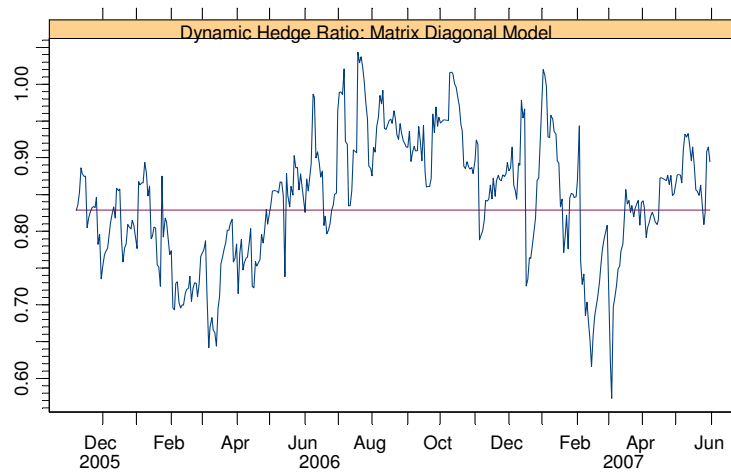


Figure 2. Dynamic Hedge Ratio – Matrix Diagonal GARCH Model



Note: The horizontal line is the constant hedge ratio (0.82) computed by OLS.
In sample period: from November 8, 2005 to May 31, 2007

Figure 3. Dynamic Hedge Ratio – CCC GARCH Model

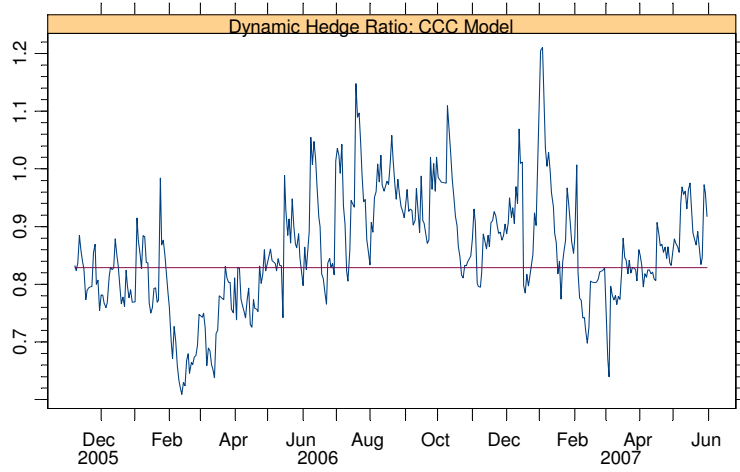


Figure 4. Dynamic Hedge Ratio - BEKK GARCH Model

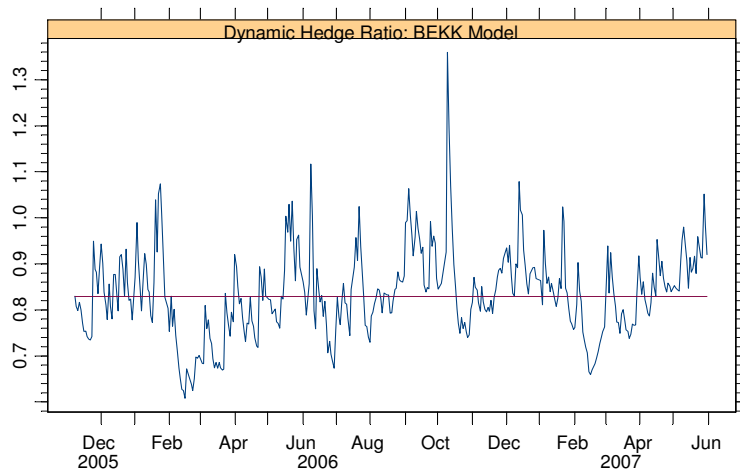
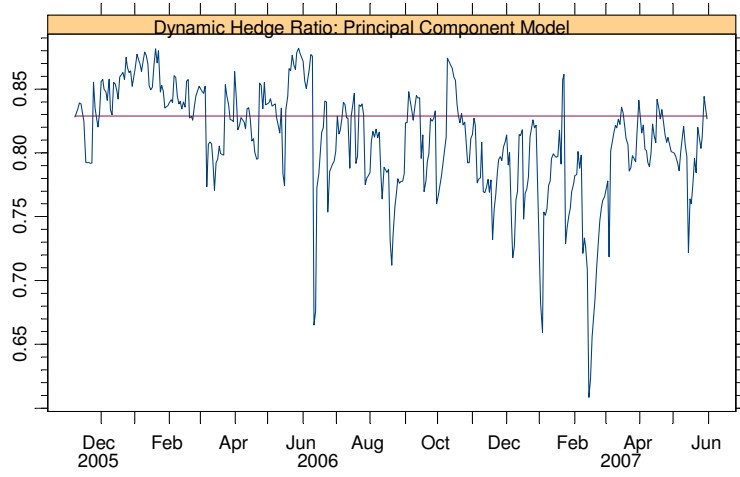


Figure 5. Dynamic Hedge Ratio – Principal Component GARCH Model



Appendix. Introduction to KOSTAR Futures

Underlying Asset	KOSDAQ STAR Index
Contract Size	STAR Index times Korean Won (KRW) 10,000
Contract Months	The four consecutive near months from the quarterly cycle(March, June, September and December)
Trading Hours	9:00 ~ 15:15 (9:00 ~ 14:50 on the last trading day)
Tick Size & Value	0.5 point, representing a value of KRW 5,000
Last Trading Day	Second Thursday of the contract month
Final Settlement Day	The following day of the last trading day
Final Settlement	Cash
Daily Price Limit	10% of the previous closing price
Position Limit	Net position of 5,000 contracts
Circuit Breakers	<p>When the lead month contract hits $\pm 6\%$ of the previous closing price for 1 minute, and the difference between the current price and the theoretical price is $\pm 3\%$ or more, the trading of all contracts are halted for the next five minutes. For the next ten minutes following the cooling-off period, orders are collected and then matched at a single price.</p> <p>Additionally, the futures and options markets are automatically suspended if the KOSDAQ market is halted. Trading in the KOSDAQ market is halted for twenty minutes if the KOSDAQ falls 10% or more from the previous closing value and the falling continues for one minute or longer.</p>

Source: Korea Exchange