6b. Consider the combination of resistors shown in the figure below. (a) Find the equivalent resistance between point a and b. [We did this yesterday...]
(b) Find the current in each resistor.

(a) The parallel combination of the 6.0 Ω and 12 Ω resistors has an equivalent resistance of
\[
\frac{1}{R_{p1}} = \frac{1}{6.0 \, \Omega} + \frac{1}{12 \, \Omega} = \frac{2 + 1}{12 \, \Omega} \quad \text{or} \quad R_{p1} = \frac{12 \, \Omega}{3} = 4.0 \, \Omega
\]
Similarly, the equivalent resistance of the 4.0 Ω and 8.0 Ω parallel combination is
\[
\frac{1}{R_{p2}} = \frac{1}{4.0 \, \Omega} + \frac{1}{8.0 \, \Omega} = \frac{2 + 1}{8.0 \, \Omega} \quad \text{or} \quad R_{p2} = \frac{8 \, \Omega}{3}
\]
The total resistance of the series combination between points a and b is then
\[
R_{ab} = R_{p1} + 5.0 \, \Omega + R_{p2} = 4.0 \, \Omega + 5.0 \, \Omega + \frac{8.0 \, \Omega}{3} = \frac{35}{3} \, \Omega
\]
(b) If \(\Delta V_{ab} = 35 \, \text{V}\), the total current from a to b is \(I_{ab} = \Delta V_{ab}/R_{ab} = 35 \, \text{V}/(35 \, \Omega/3) = 3.0 \, \text{A}\) and the potential differences across the two parallel combinations are
\[
\Delta V_{p1} = I_{ab}R_{p1} = (3.0 \, \text{A})(4.0 \, \Omega) = 12 \, \text{V} \quad \text{and} \quad \Delta V_{p2} = I_{ab}R_{p2} = (3.0 \, \text{A})(\frac{8.0 \, \Omega}{3}) = 8.0 \, \text{V}
\]
so the individual currents through the various resistors are:
\[
I_1 = \Delta V_{p1}/12 \, \Omega = \frac{1.0}{12} \, \text{A} \quad ; \quad I_6 = \Delta V_{p1}/6.0 \, \Omega = \frac{2.0}{6.0} \, \text{A} \quad ; \quad I_5 = I_{ab} = 3.0 \, \text{A} \quad ;
\]
\[
I_8 = \Delta V_{p2}/8.0 \, \Omega = \frac{1.0}{8.0} \, \text{A} \quad ; \quad \text{and} \quad I_4 = \Delta V_{p2}/4.0 \, \Omega = \frac{2.0}{4.0} \, \text{A}
\]
16. (a) Find the current in each resistor in the figure below by using the rules for resistors in series and parallel.

![Circuit Diagram]

(a) The equivalent resistance of the parallel combination between points $b$ and $e$ is

$$\frac{1}{R_{be}} = \frac{1}{12 \, \Omega} + \frac{1}{24 \, \Omega} \quad \text{or} \quad R_{be} = 8.0 \, \Omega$$

The total resistance between points $a$ and $e$ is then

$$R_{ae} = R_{ab} + R_{be} = 6.0 \, \Omega + 8.0 \, \Omega = 14 \, \Omega$$

The total current supplied by the battery (and also the current in the $6.0 \, \Omega$ resistor) is

$$I_{total} = I_e = \frac{\Delta V_{ae}}{R_{ae}} = \frac{42 \, \text{V}}{14 \, \Omega} = 3.0 \, \text{A}$$

The potential difference between points $b$ and $e$ is

$$\Delta V_{be} = R_{be} I_{total} = (8.0 \, \Omega)(3.0 \, \text{A}) = 24 \, \text{V}$$

so

$$I_{12} = \frac{\Delta V_{be}}{R_{12e}} = \frac{24 \, \text{V}}{12 \, \Omega} = 2.0 \, \text{A} \quad \text{and} \quad I_{24} = \frac{\Delta V_{be}}{R_{24e}} = \frac{24 \, \text{V}}{24 \, \Omega} = 1.0 \, \text{A}$$

(b) Write three independent equations for the three currents using Kirchhoff’s Laws; one with the node rule, a second using a loop through the battery, the $6.0 \, \Omega$ resistor and the $24.0 \, \Omega$ resistor, and the third using the loop rule through the $12.0 \, \Omega$ and $24.0 \, \Omega$ resistors. Solve to these equations to check the answers found in part (a).

(b) Applying the junction rule at point $b$ yields

$$I_6 - I_{12} - I_{24} = 0 \quad \text{[1]}$$

Using the loop rule on loop $abdea$ gives

$$-42 - 6I_6 - 24I_{24} = 0 \quad \text{or} \quad I_6 = 7.0 - 4I_{24} \quad \text{[2]}$$

and using the loop rule on loop $bcedb$ gives

$$-12I_{12} + 24I_{24} = 0 \quad \text{or} \quad I_{12} = 2I_{24} \quad \text{[3]}$$


$$7I_{24} = 7.0 \quad \text{or} \quad I_{24} = 1.0 \, \text{A}$$


$$I_6 = 3.0 \, \text{A} \quad \text{and} \quad I_{12} = 2.0 \, \text{A}$$
17. The figure below shows a circuit diagram. Determine (a) the current, (b) the potential of the wire A relative to ground, and (c) the voltage drop across the 1500 Ω resistor.

Going counterclockwise around the upper loop, applying Kirchhoff’s loop rule, gives

\[ +15.0 \text{ V} - (7.00)I_1 - (5.00)(2.00 \text{ A}) = 0 \]

or

\[ I_1 = \frac{15.0 \text{ V} - 10.0 \text{ V}}{7.00 \Omega} = 0.714 \text{ A} \]

From Kirchhoff’s junction rule, \( I_1 + I_2 = 2.00 \text{ A} \) = 0

so \( I_2 = 2.00 \text{ A} - I_1 = 2.00 \text{ A} - 0.714 \text{ A} = 1.29 \text{ A} \)

Going around the lower loop in a clockwise direction gives

\[ +E - (2.00)I_2 - (5.00)(2.00 \text{ A}) = 0 \]

or

\[ E = (2.00 \Omega)(1.29 \text{ A}) + (5.00 \Omega)(2.00 \text{ A}) = 12.6 \text{ V} \]

22abc. Four resistors are connected to a battery with a terminal voltage of 12 V as shown in the figure below. (a) How would you reduce the circuit to an equivalent single resistor connected to the battery? Use this procedure to find the equivalent resistance of the circuit. (b) Find the current delivered by the battery to this equivalent resistance. (c) Determine the power delivered by the battery.
(a) The 30.0 Ω and 50.0 Ω resistors in the upper branch are in series, and add to give a total resistance of \( R_{\text{upper}} = 80.0 \, \Omega \) for this path. This 80.0 Ω resistance is in parallel with the 80.0 Ω resistance of the middle branch, and the rule for combining resistors in parallel yields a total resistance of \( R_{\text{ab}} = 40.0 \, \Omega \) between points \( a \) and \( b \). This resistance is in series with the 20.0 Ω resistor, so the total equivalent resistance of the circuit is
\[
R_{\text{eq}} = 20.0 \, \Omega + R_{\text{ab}} = 20.0 \, \Omega + 40.0 \, \Omega = 60.0 \, \Omega
\]

(b) The current supplied to this circuit by the battery is
\[
I_{\text{total}} = \frac{\Delta V}{R_{\text{eq}}} = \frac{12 \, \text{V}}{60.0 \, \Omega} = 0.20 \, \text{A}
\]

(c) The power delivered by the battery is
\[
P_{\text{total}} = R_{\text{eq}} I_{\text{total}}^2 = (60.0 \, \Omega)(0.20 \, \text{A})^2 = 2.4 \, \text{W}
\]

32. An uncharged capacitor and a resistor are connected in series to a source of emf. If \( V = 9.00 \, \text{V} \), \( C = 20.0 \, \mu\text{F} \), and \( R = 100 \, \Omega \),
(a) find the time constant of the circuit,
(b) the maximum charge on the capacitor, and
(c) the charge on the capacitor after one time constant. [Find the voltage on the capacitor first.]

(a) \( \tau = RC = (100 \, \Omega)(20.0 \times 10^{-6} \, \text{F}) = 2.00 \times 10^{-3} \, \text{s} = 2.00 \, \text{ms} \)

(b) \( Q_{\text{max}} = CE = (20.0 \times 10^{-6} \, \text{F})(9.00 \, \text{V}) = 1.80 \times 10^{-5} \, \text{C} = 180 \, \mu\text{C} \)

(c) \[ Q = Q_{\text{max}}(1 - e^{-\tau/\tau}) = Q_{\text{max}}(1 - e^{-1}) = Q_{\text{max}} \left(1 - \frac{1}{e}\right) = 114 \, \mu\text{C} \]
34. A series combination of a 12 kΩ resistor, and an unknown capacitor is connected to a 12 V battery. One second after the circuit is completed, the voltage across the capacitor is 10.0 V. Determine the capacitance of the capacitor. [Hint: how many time constants is 1 second?]

The charge on the capacitor at time \( t \) is \( Q = Q_{\text{max}} \left( 1 - e^{-\frac{t}{\tau}} \right) \), where

\[
Q = C\left( \Delta V \right) \quad \text{and} \quad Q_{\text{max}} = CE.
\]

Thus, \( \Delta V = E \left( 1 - e^{-\frac{t}{\tau}} \right) \) or \( e^{-\frac{t}{\tau}} = 1 - \left( \frac{\Delta V}{E} \right) \)

We are given that \( E = 12 \text{ V} \), and at \( t = 1.0 \text{ s} \), \( \Delta V = 10 \text{ V} \)

Therefore, \( e^{-1.0/\tau} = 1 - \frac{10}{12} = \frac{12 - 10}{12} = \frac{1}{6.0} \) or \( e^{1.0/\tau} = 6.0 \)

Taking the natural logarithm of each side of the equation gives

\[
\frac{1.0 \text{ s}}{\tau} = \ln \left( 6.0 \right) \quad \text{or} \quad \tau = \frac{1.0 \text{ s}}{\ln \left( 6.0 \right)} = 0.56 \text{ s}
\]

Since the time constant is \( \tau = RC \), we have

\[
C = \frac{\tau}{R} = \frac{0.56 \text{ s}}{12 \times 10^3 \text{ Ω}} = 4.7 \times 10^{-5} \text{ F} = 47 \mu\text{F}
\]