In the “Evaluating a Claim of Hearing Loss” activity, we encountered several basic examples that required us to think statistically in order to investigate a question of interest. Before we move on to slightly more complex examples, we will discuss some basic definitions that will be used throughout the semester.

**SOME BASIC DEFINITIONS**

- **Statistics:**
- **Categorical (or qualitative) data:** Measurements that are classified into one of a group of categories.
- **Numerical (or quantitative) data:** Measurements that are recorded on a naturally occurring numerical scale.

Most of what we’ll be doing in this course centers on trying to understand a set of information. This set of information is from a . . .

- **Population:** The complete collection of ALL elements that are of interest for a given problem.
  
  The population is often so big that obtaining all information about its elements is either difficult or impossible. So, we work with a more manageable set of data that we obtain from a . . .

- **Sample:** A subcollection of elements drawn from a population.

- **Observation:** The collection of measurements from a particular unit in a sample.

- **Variable:** Any measurable characteristic of an observation.

**Example 1.1: Alleged Hearing Loss**

Consider once again the example presented in an article by Pankratz, Fausti, and Peed titled “A Forced-Choice Technique to Evaluate Deafness in the Hysterical or Malingering Patient.” Recall that the subject was correct on 36 out of 100 trials when he was asked to identify whether the tone played with either the red or the blue lightbulb.

Identify the following in the context of this example:

- Variable of interest:
- Data type:
- Population of interest:
- Sample:

Recall that we carried out a simulation study to determine whether this patient who was suspected of malingering had obtained too few correct answers. The results of the simulation study indicate what outcomes we expect from a guessing subject:

![Simulation Study Results]

**Question:** Based on the results of this simulation study, do you believe the patient’s outcome of 36 correct out of 100 was consistent with guessing, or do these results indicate that he may have been answering incorrectly on purpose in order to mislead the researchers into believing he was hearing impaired?
A Statistical Investigation for Example 1.1 from Start to Finish

Research Question
Is there statistical evidence that the subject is intentionally answering incorrectly in order to convince his doctors that he can’t hear?

Design Study/Collect Data
The researchers presented the subject with 100 trials of the red/blue lightbulb experiment. They kept track of the number of trials in which he gave a correct answer.

Explore/Summarize Data
Of the 100 trials, he answered only 36 correct (36%).

Draw Appropriate Inferences Beyond the Sample
The subject’s outcome (only 36 correct out of 100 trials) was much lower than what was expected if he were truly deaf, and it was shown to be very unlikely to occur by chance if he were truly deaf. Even though these 100 trials provide just a sample of the subject’s overall behavior, the statistical evidence is strong enough to indicate that the actual long-run probability of the subject answering correctly is much lower than we would expect if he were truly deaf (i.e., there is evidence he is not deaf).

Example 1.2: Helper vs. Hinderer?

In a study reported in a November 2007 issue of Nature, researchers investigated whether infants take into account an individual’s actions towards others in evaluating that individual as appealing or aversive, perhaps laying the foundation for social interaction (Hamlin, Wynn, and Bloom, 2007). In one component of the study, sixteen 10-month-old infants were shown a “climber” character (a piece of wood with “google” eyes glued onto it) that could not make it up a hill in two tries. Then they were shown two scenarios for the climber’s next try, one where the climber was pushed to the top of the hill by another character (“helper”) and one where the climber was pushed back down the hill by another character (“hinderer”). The infant was alternately shown these two scenarios several times. Then the child was presented with both pieces of wood (the helper and the hinderer) and asked to pick one to play with. The color and shape and order (left/right) of the toys were varied and balanced out among the 16 infants.

References:

Research Question: Do 10-month-old infants show a preference for the helper toy over the hinderer toy?

Questions:

1. Why was it important for the researchers to balance out the color, shape, and order of the toys across the study? For example, how would the study results have been affected if the researchers always made the helper toy a blue circle and the hinderer a yellow triangle?

2. Identify the following in the context of this example:
   - Variable of interest:
   - Data type:
   - Population of interest:
   - Sample:

3. Recall that this study involves 16 infants. If the population of all 10-month-old infants has no real preference for one toy over the other, how many infants do you expect to choose the helper toy? Explain.

4. Suppose that 10 out of 16 infants choose the helper toy (62.5%). Since this value is higher than 50%, a researcher argues that these data show that the majority of all 10-month-old infants would choose the helper toy. What is wrong with their reasoning?

Once again, the key question is how to determine whether the study’s result is surprising under the assumption that there is no real preference for one toy over the other in the population of all 10-month-old infants. To answer this, we will simulate the process of 16 infants simply choosing a toy at random, over and over again. Each time we simulate the process, we’ll keep track of how many infants out of the 16 chose the helper toy (note that you could also keep track of the number that chose the hinderer toy). Once we’ve repeated this process a large number of times, we’ll have a pretty good sense for what outcomes would be very surprising, somewhat surprising, or not so surprising if the population of all 10-month-old infants has no real preference.
Carry out the Tinkerplots simulation. Note that you should consider the following questions when designing your simulation study:

- What are the two possible outcomes on each of the trials? Change the values on your spinner accordingly.

- What is the probability that each outcome occurs under the assumption that the population of all 10-month-old infants has no real preference for either toy? Change your spinner accordingly.

- Be sure to change the **Draw** value to 1 since only one infant is choosing a toy at a time.

- How many infants were used in this study? Keep this value in mind when setting the **Repeat** value.

Carry out the simulation study 1000 times overall, keeping track of the number of infants that choose the helper toy in each of the simulated experiments. Sketch in your results below:
Questions:

5. What does each dot on this plot represent?

6. Suppose that in the actual study 10 out of 16 infants chose the helper toy. Would this convince you that the majority of the population of all 10-month-old infants had a preference for the helper toy? Why or why not?

7. **The actual study results are as follows: 14 out of 16 infants chose the helper toy.** Mark this on the axis above the results of your simulations study. Based on this statistical investigation, what should the researchers conclude? Recall that their research question was stated as follows: Do 10-month-old infants tend to prefer the helper toy over the hinderer toy?

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**A Statistical Investigation for Example 1.2 from Start to Finish**

<table>
<thead>
<tr>
<th><strong>Research Question</strong></th>
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<tbody>
<tr>
<td>Do 10-month-old infants show a preference for the helper toy over the hinderer toy?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Design Study/Collect Data</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>The study was conducted as described on page 3. Researchers kept track of which toy was chosen by the infant (helper or hinderer).</td>
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</table>

<table>
<thead>
<tr>
<th><strong>Explore/Summarize Data</strong></th>
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</thead>
<tbody>
<tr>
<td>Of the 16 infants, 14 chose the helper toy (87.5%).</td>
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</table>

<table>
<thead>
<tr>
<th><strong>Draw Appropriate Inferences Beyond the Sample</strong></th>
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<tbody>
<tr>
<td>The study’s outcome (14 of 16 infants choosing the helper) indicated that a strong majority of the sample showed a preference for the helper toy. Furthermore, the outcome was shown to be very unlikely to occur by chance if there were no real preference for one toy over the other in the population of all 10-month-olds. This provides strong statistical evidence that there is a preference for the helper toy in the population of all 10-month-olds, as well.</td>
</tr>
</tbody>
</table>

**Big idea:** Even though the 16 infants studied are just a sample of the population of all 10-month-olds, we can draw a conclusion about the population of all 10-month-olds.
Example 1.3: Are Women Passed Over for Managerial Training?

This example involves possible discrimination against female employees. Suppose a large supermarket chain occasionally selects employees to receive management training. A group of female employees has claimed that they are less likely than male employees of similar qualifications to be chosen for this training.

The large employee pool that can be tapped for management training is 60% female and 40% male; however, since the management program began, 9 of the 20 employees chosen for management training were female (only 45%). Do the female employees have a valid statistical argument that they are being discriminated against?

Research Question: Is there statistical evidence for gender discrimination against females?

Questions:

1. Identify the following in the context of this example:

   - Variable of interest:
   - Data type:
   - Population of interest:
   - Sample:

2. If the selection process was unbiased, how many of the 20 employees selected for management do you expect to be women? Explain.

Once again, the key question is how to determine whether the result is surprising under the assumption that the selection process is unbiased. To answer this, we will simulate the process of an unbiased selection process, over and over again. Each time we simulate the process, we’ll keep track of how many of the 20 employees selected for management were female. Once we’ve repeated this process a large number of times, we’ll have a pretty good sense for what outcomes would be very surprising, somewhat surprising, or not so surprising if there was no discrimination in the selection process.
Carry out the Tinkerplots simulation. Note that you should consider the following questions when designing your simulation study:

- What are the two possible outcomes on each of the trials? Change the values on your spinner accordingly.

- What is the probability that each outcome occurs under the assumption that there is no gender discrimination in the selection process? Change your spinner accordingly.

- Be sure to change the Draw value to 1 since only one employee was chosen at a time.

- How many subjects were there in this study? Keep this value in mind when setting the Repeat value.

Carry out the simulation study 1000 times overall, keeping track of the number of the employees chosen for management that were female on each of the simulated experiments. Sketch in your results below:
Questions:

3. What does each dot on this plot represent?

4. Recall that since the management program began, only 9 of the 20 employees chosen for management training were female. Does this outcome convince you that the selection process is biased against women? Why or why not?

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A Statistical Investigation for Example 1.3 from Start to Finish

<table>
<thead>
<tr>
<th>Research Question</th>
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</thead>
<tbody>
<tr>
<td>Is there statistical evidence for gender discrimination against females?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Design Study/Collect Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Researchers kept track of the gender of employees chosen for management since the inception of the management program. began, 9 of the 20 employees chosen for management training were female (only 45%).</td>
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</table>

<table>
<thead>
<tr>
<th>Explore/Summarize Data</th>
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<tbody>
<tr>
<td>Of the 20 employees chosen, 9 were female (45%).</td>
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<table>
<thead>
<tr>
<th>Draw Appropriate Inferences Beyond the Sample</th>
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<tbody>
<tr>
<td>The study’s outcome (9 of 20 employees chosen were female) was a bit lower than what was expected if the company were not discriminating. However, the outcome was shown to be somewhat likely to occur by chance even if the company was not discriminating. Even though the 20 employees chosen so far are just a sample of all employees the company might ever choose for management, we can still make a conclusion about the true long-run probability of a company choosing a female for management. In this case, the statistical evidence is not strong enough to indicate that the company is discriminating against females.</td>
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</table>
Example 1.4: Font Preferences

Researchers carried out a marketing field study in order to study preferences of potential consumers in the U.S. They used silver cardboard boxes to contain chocolate truffles in a forced choice task. All of the box tops were decorated in the same way, and a white label was attached to each bearing the name “Indulgence” in either Signet font or Salem font. The text on each label was approximately equal-sized. For each of the 40 subjects in the study, one box labeled with the Signet font and another box labeled with the Salem font were placed on a tray, and the subject was simply asked to choose a truffle from one of the two boxes that were on the tray in front of them.

<table>
<thead>
<tr>
<th>Font Style</th>
<th>Signet</th>
<th>Salem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indulgence</td>
<td>Indulgence</td>
<td></td>
</tr>
</tbody>
</table>

The researchers aren’t sure which font is more appropriate for the label and simply want to know whether the majority of all consumers will choose the truffles with one font more than the other. In the sample of 40 subjects, 30 chose to take a truffle from the box that had Signet font.

**Research Question:** Do the majority of consumers have a preference for one font over the other?
Questions:

1. Identify the following in the context of this example:
   - Variable of interest:
   - Data type:
   - Population of interest:
   - Sample:

2. If there was no preference in the population, how many of the 40 consumers do you expect to choose Signet font? Explain.

To gain an understanding of what outcomes we expect to see if there is no real preference in the population of all consumers, we will simulate this experiment under the condition that there is no preference for one font over the other. Carry out the Tinkerplots simulation. Note that you should consider the following questions when designing your simulation study:

- What are the two possible outcomes on each of the trials? Change the values on your spinner accordingly.
- What is the probability that each outcome occurs under the assumption that there is no preference in the population? Change your spinner accordingly.
- Be sure to change the Draw value to 1.
- How many subjects were there in this study? Keep this value in mind when setting the Repeat value.

Carry out the simulation study 1000 times overall, keeping track of the number that choose Signet on each of the simulated experiments. Sketch in your results on the following graph:
Questions:

3. What does each dot on this plot represent?

4. In the actual study, 30 of the 40 selected the Signet font. Does this outcome convince you that there is a preference for one font over the other? Why or why not?
### A Statistical Investigation for Example 1.4 from Start to Finish

<table>
<thead>
<tr>
<th><strong>Research Question</strong></th>
<th>Do the majority of consumers have a preference for one font over the other?</th>
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</thead>
<tbody>
<tr>
<td><strong>Design Study/Collect Data</strong></td>
<td>The study was conducted as described on page 10. Researchers kept track of which font was chosen by the consumer (Signet or Salem).</td>
</tr>
<tr>
<td><strong>Explore/Summarize Data</strong></td>
<td>Of the 40 consumers, 30 chose the Signet font (75%).</td>
</tr>
<tr>
<td><strong>Draw Appropriate Inferences Beyond the Sample</strong></td>
<td>The study’s outcome (30 of 40 choosing the Signet font) was much higher than what was expected if there were no real font preference in the population. Furthermore, the outcome was shown to be very unlikely to occur by chance if there were no real preference for one font over the other in the population of all consumers. This provides strong statistical evidence that there is a preference for one font over the other in the population of consumers.</td>
</tr>
</tbody>
</table>

**Big idea:** Even though the 40 consumers studied are just a sample of the population of all consumers, we can draw a conclusion about the population of all consumers.