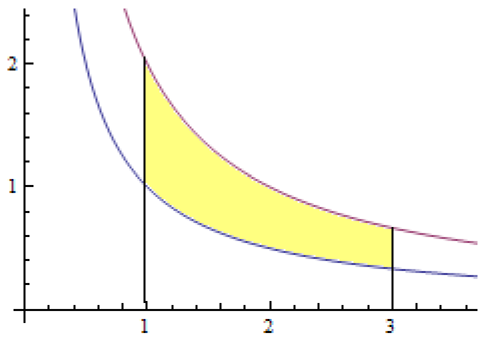
Practice Problems on Volumes of Solids of Revolution

Find the volume of each of the following solids of revolution obtained by rotating the indicated regions.

a. Bounded by $y = 1/x$, $y = 2/x$, and the lines $x = 1$ and $x = 3$ rotated about the x-axis.



Disk: $V = \pi \int_1^3 \{(2/x)^2 - (1/x)^2\}\, dx = 2\pi$

b. The region in the preceding problem rotated about the line $y = -1$.

Disk: $V = \pi \int_1^3 \{(2/x + 1)^2 - (1/x + 1)^2\}\, dx = \pi(2 + 2 \operatorname{Ln} 3)$

c. Bounded by $y = 1/x$, $y = 2/x$, and the lines $x = 1$ and $x = 3$ rotated about the y-axis. (Previous region)

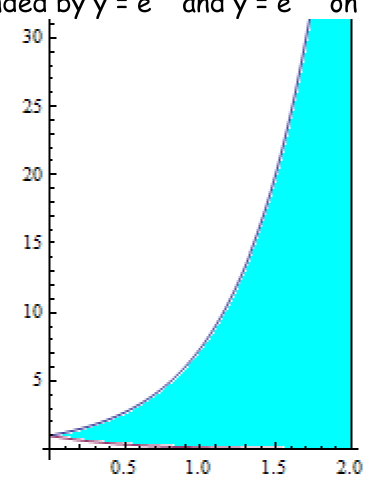
Disk: $V = \pi \int_1^2 \{(2/y)^2 - 1^2\}\, dy + \pi \int_{2/3}^1 \{(2/y)^2 - (1/y)^2\}\, dy + \pi \int_{1/3}^{2/3} \{3^2 - (1/y)^2\}\, dy = 4\pi$

Cylindrical Shell: $V = 2\pi \int_1^3 x\{2/x - 1/x\}\, dx = 4\pi$

d. Use the Cylindrical Shell Method to find the volume of the solid obtained by rotating the region bounded by $y = \sqrt{1 - x^2}$, the x-axis, and the y-axis in the first quadrant rotated about the y-axis.

$V = 2\pi \int_0^1 x \sqrt{1 - x^2}\, dx = -\frac{2}{3}\pi (1 - x^2)^{3/2} \Big|_0^1 = 2\pi/3$

e. Bounded by $y = e^{2x}$ and $y = e^{-2x}$ on $[0, 2]$ rotated about the x-axis.



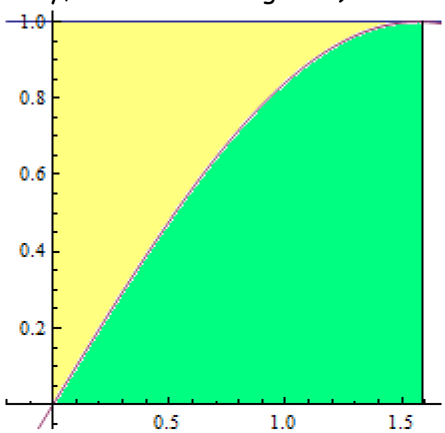
Disk: $V = \pi \int_0^2 \{(e^{2x})^2 - (e^{-2x})^2\}\, dx = \dfrac{e^8 + e^{-8} - 2}{4}\pi$

f. The region in the preceding problem rotated about the y-axis. (Set up the integral only, but do not integrate.)

Cylindrical Shell: $V = 2\pi \int_0^2 x\{e^{2x} - e^{-2x}\}\, dx$

Disk: $V = \pi \int_1^{e^4} \{2^2 - (\frac{1}{2}\text{Ln } y)^2\}\, dy + \pi \int_{e^{-4}}^1 \{2^2 - (-\frac{1}{2}\text{Ln } y)^2\}\, dy$

g. The region above y = sin x and below y = 1 on [0, π/2] (yellow region below) rotated about the x-axis. (Set up the integral only, but do not integrate.)



Disk: $V = \pi \int_0^{\pi/2} [1 - \sin^2 x]\, dx$

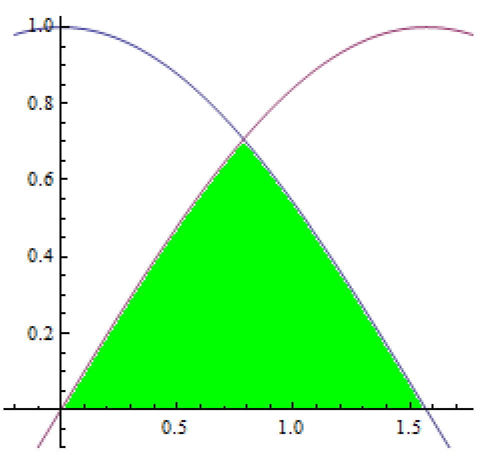
Cylindrical Shell: $V = 2\pi \int_0^1 x^*\arcsin x\, dx$

h. The region below y = sin x and above the x-axis on [0, π/2] (green region in the preceding problem) rotated about the y-axis. (Set up the integral only, but do not integrate.)

Cyndrical Shell: $V = 2\pi \int_{-\pi/2}^{\pi/2} x\,\{\sin x\}\, dx$

Disk: $V = \pi \int_0^1 \{(\pi/2)^2 - (\arcsin y)^2\}\, dy$

i. The region below y = sin x and y = cos x but above the x-axis on [0, π/2]. (Set up the integral only, but do not integrate.)



Disk: $\pi \int_0^{\pi/4} \{\sin^2 x\}\, dx + \pi \int_{\pi/4}^{\pi/2} \{\cos^2 x\}\, dx$

Cylindrical Shell: $2\pi \int_0^{\sqrt{2}/2} y*\{\arccos y \ - \ \arcsin y\}\, dy$

j. The region in the preceding problem rotated about the y-axis. (Set up the integral only, but do not integrate.)

Disk: $\pi \int_0^{\sqrt{2}/2} \{ (\pi/4)^2 \ - \ (\arccos y)^2 \}\, dy + \pi \int_0^{\sqrt{2}/2} \{ (\arcsin y)^2 - (\pi/4)^2 \}\, dy$

Cylindrical Shell: $2\pi \int_0^{\pi/4} x \sin x\, dx + 2\pi \int_{\pi/4}^{\pi/2} x \cos x\, dx$

k. The region bounded by $y = 4 - x^2$ and $y = x^4 + 2$ rotated about the x-axis.

Disk: $A = \int_{-1}^{1} \{(4 - x^2)^2 - (x^4 + 2)^2)\, dx = \dfrac{776\pi}{45}$