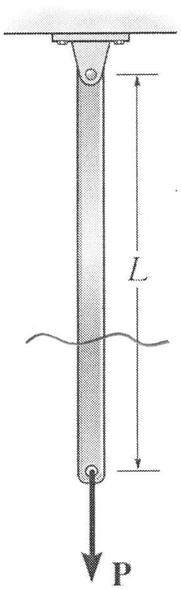
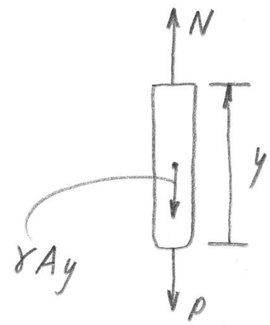
CME 260 – Mechanics of Materials
Exam #3 (02/23/2022)

Tech ID or Star ID: *Grading*

Do one of the two problems shown below (the second problem is on the back).
<u>Show your work</u> (you will not receive any credit if all you have is a final answer, right or wrong).

1. The bar shown has a length L and cross-sectional area A. Determine the change in the length of the bar due to the force P and its own weight. The material has a specific weight γ (weight/volume) and a modulus of elasticity E. (your answer is to be given in terms of the variables provided).





$$\sum F_y = 0, \quad N - \gamma A y - P = 0$$

$$N = \gamma A y + P$$

$$\delta = \int_0^L \frac{N(y)\,dy}{AE}$$

$$= \int_0^L \frac{(\gamma A y + P)\,dy}{AE}$$

$$= \frac{1}{AE}\int_0^L (\gamma A y + P)\,dy$$

$$= \frac{1}{AE}\left[\int_0^L \gamma A y\,dy + \int_0^L P\,dy\right]$$

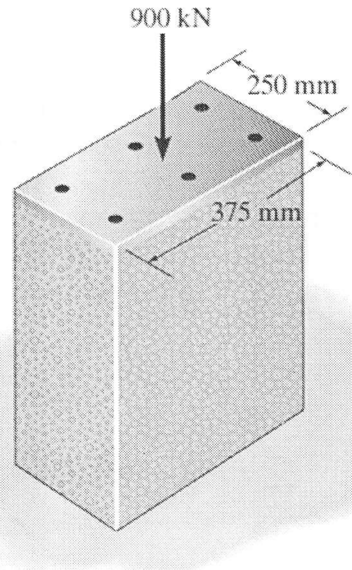
$$= \frac{1}{AE}\left[\gamma A \frac{y^2}{2}\Big|_0^L + Py\Big|_0^L\right]$$

$$= \frac{1}{AE}\left[\frac{\gamma A L^2}{2} + PL\right]$$

$$= \boxed{\frac{\gamma L^2}{2E} + \frac{PL}{AE}}$$

(5 pts) (5 pts)

2. The concrete column is reinforced with six steel reinforcing rods. Each reinforcing rod has a diameter of 20 mm. The column is subjected to a 900 kN axial force as shown. E_{st} = 200 GPa and E_c = 25 GPa. Determine:

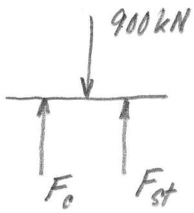


(a) The stress in the concrete.
(b) The stress in the steel.

$A_{st} = (6)\frac{\pi}{4}(0.020\,m)^2 = 1.885\times10^{-3}\,m^2$

$A_c = (0.375\,m)(0.250\,m) - A_{st}$ (1 pt)
$= 9.187\times10^{-2}\,m^2$

equilibrium:



$\Sigma F_y = 0, \; -900\,kN + F_c + F_{st} = 0$

$F_c + F_{st} = 900\,kN$ (2 pts)

compatibility:

$\delta_c \{\text{- - - - -}\} \delta_{st}$ $\quad \delta_c = \delta_{st}$ (2 pts)

$$\frac{F_c L}{A_c E_c} = \frac{F_{st} L}{A_{st} E_{st}}$$

$$\frac{F_c}{A_c E_c} = \frac{900\,kN - F_c}{A_{st} E_{st}} \longrightarrow \frac{F_c}{A_c E_c} + \frac{F_c}{A_{st} E_{st}} = \frac{900\,kN}{A_{st} E_{st}}$$

$$F_c\left(\frac{A_{st}E_{st} + A_c E_c}{(A_c E_c)(A_{st} E_{st})}\right) = \frac{900\,kN}{A_{st} E_{st}}$$

$$F_c = \frac{900\,kN}{\cancel{A_{st} E_{st}}}\left(\frac{(A_c E_c)\cancel{(A_{st} E_{st})}}{A_{st} E_{st} + A_c E_c}\right)$$

$$= \frac{(900\,kN)(9.187\times10^{-2}\,m^2)(25\times10^6\,\frac{kN}{m^2})}{(1.885\times10^{-3})(200\times10^6\,\frac{kN}{m^2}) + (9.187\times10^{-2}\,m^2)(25\times10^6\,\frac{kN}{m^2})}$$

$= 773.1\,kN$ (3 pts)

$F_{st} = 900\,kN - 773.1\,kN = 126.9\,kN$ (1 pt)

$$\sigma_{st} = \frac{126.9\,kN}{1.885\times10^{-3}\,m^2} = \boxed{67{,}321\,kPa \text{ or } 67.3\,MPa}$$

(1 pt)

$$\sigma_c = \frac{773.1\,kN}{9.187\times10^{-2}\,m^2} = \boxed{8{,}415.2\,kPa \text{ or } 8.42\,MPa}$$