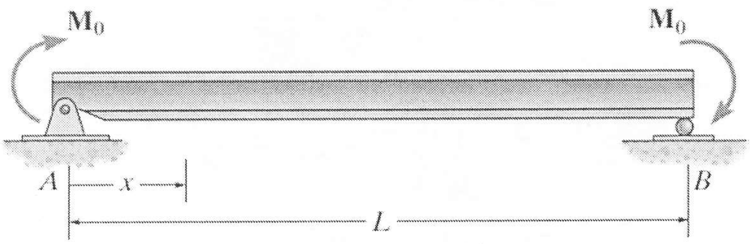
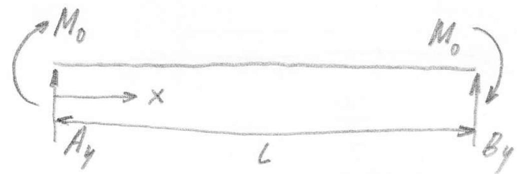
Tech ID or Star ID: Grading

Do one of the two problems shown below (the second problem is on the back).
Show your work (you will not receive any credit if all you have is a final answer, right or wrong).

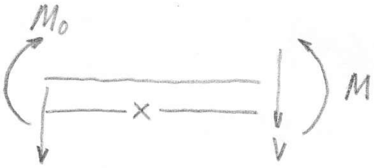
(1) Determine the equation of the elastic curve for the beam using the x coordinate. What is the slope at A and the deflection at the center of the beam? EI is constant.





$\Sigma M_B = 0, \quad -LA_y - M_o - M_o = 0$

$A_y = -\frac{2M_o}{L}$ (1pt)



$A_y = \frac{2M_o}{L}$

$\Sigma M_x = 0, \quad \frac{2M_o}{L}x - M_o + M = 0$

$M = -\frac{2M_o}{L}x + M_o$ (1pt)

$$\frac{d^2v}{dx^2} = \frac{1}{EI}\left(-\frac{2M_o}{L}x + M_o\right)$$

$$\frac{dv}{dx} = \frac{1}{EI}\left(-\frac{M_o}{L}x^2 + M_o x + C_1\right) \quad (1pt)$$

$$v = \frac{1}{EI}\left(-\frac{M_o}{3L}x^3 + \frac{M_o}{2}x^2 + C_1 x + C_2\right) \quad (1pt)$$

@ $x=0, v=0 \longrightarrow C_2 = 0$ (1pt)

@ $x=L, v=0$

$$0 = \frac{1}{EI}\left(-\frac{M_o}{3L}L^3 + \frac{M_o}{2}L^2 + C_1 L\right)$$

$$C_1 \cancel{L} = \frac{M_o L^{\cancel{2}}}{3} - \frac{M_o L^{\cancel{2}}}{2}$$

$$C_1 = -\frac{M_o L}{6} \quad (1pt)$$

$$\frac{dv}{dx} = \frac{1}{EI}\left(-\frac{M_o}{L}x^2 + M_o x - \frac{M_o L}{6}\right)$$

$$= \frac{M_o}{6EIL}\left(-6x^2 + 6Lx - L^2\right) \quad (1pt)$$

$$v = \frac{1}{EI}\left(-\frac{M_o}{3L}x^3 + \frac{M_o}{2}x^2 - \frac{M_o L}{6}x\right)$$

$$= \boxed{\frac{M_o}{6EIL}\left(-2x^3 + 3Lx^2 - L^2 x\right)} \quad (1pt)$$

slope @ A, $x=0$

$$\frac{dv}{dx} = \frac{M_o}{6EIL}\left(-6(0)^2 + 6L(0) - L^2\right)$$

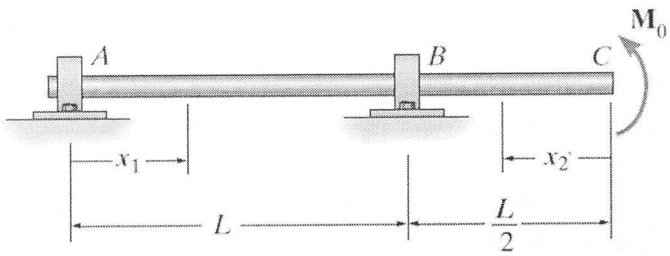
$$= -\frac{M_o L^{\cancel{2}}}{6EI\cancel{L}} = \boxed{-\frac{M_o L}{6EI}} \quad (1pt)$$

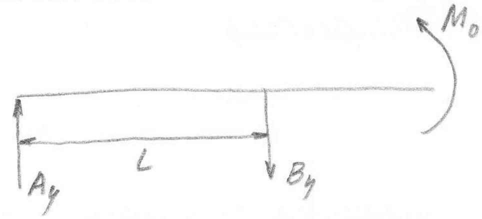
deflection @ center, $x = \frac{L}{2}$

$$v = \frac{M_o}{6EIL}\left(-2\left(\frac{L}{2}\right)^3 + 3L\left(\frac{L}{2}\right)^2 - L^2\left(\frac{L}{2}\right)\right)$$

$$= \frac{M_o}{6EIL}\left(-\frac{L^3}{4} + \frac{3L^3}{4} - \frac{L^3}{2}\right) = \boxed{0} \quad (1pt)$$

(2) Determine the equations of the elastic curve for the shaft using the x_1 and x_2 coordinates. What is the slope at A and the deflection at C? EI is constant. The supports at A and B prevent translation of the shaft in the vertical direction but rotation can occur.

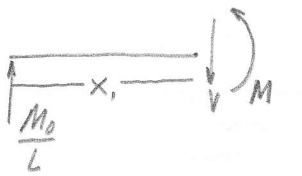




$$\Sigma M_B = 0, \quad -LA_y + M_o = 0$$

$$A_y = \frac{M_o}{L}$$

$0 \le x_1 < L$



$$\Sigma M_{x_1} = 0, \quad -\frac{M_o}{L} x_1 + M = 0$$

$$M = \frac{M_o}{L} x_1 \quad (1\,pt)$$

$$\frac{d^2 v_1}{dx_1^2} = \frac{1}{EI}\left(\frac{M_o x_1}{L}\right)$$

$$\frac{dv_1}{dx_1} = \frac{1}{EI}\left(\frac{M_o x_1^2}{2L} + C_1\right)$$

$$v_1 = \frac{1}{EI}\left(\frac{M_o x_1^3}{6L} + C_1 x_1 + C_2\right)$$

$@\ x_1 = 0,\ v_1 = 0 \longrightarrow C_2 = 0 \quad (1\,pt)$

$@\ x_1 = L,\ v_1 = 0$

$$0 = \frac{1}{EI}\left(\frac{M_o L^{\cancel{3}\,2}}{6\cancel{L}} + C_1 L\right)$$

$$C_1 L = -\frac{M_o L^2}{6}$$

$$C_1 = -\frac{M_o L}{6} \quad (1\,pt)$$

$$\frac{dv_1}{dx_1} = \frac{1}{EI}\left(\frac{M_o x_1^2}{2L} - \frac{M_o L}{6}\right)$$

$$= \frac{M_o}{6EIL}\left(3x_1^2 - L^2\right) \quad (1\,pt)$$

$$v_1 = \frac{1}{EI}\left(\frac{M_o x_1^3}{6L} - \frac{M_o L}{6} x_1\right)$$

$$= \boxed{\frac{M_o}{6EIL}\left(x_1^3 - L^2 x_1\right)} \quad (1\,pt)$$

$@\ A,\ x_1 = 0$

$$\frac{dv_1}{dx_1} = \frac{M_o}{6EIL}\left(3(0)^2 - L^2\right)$$

$$= \boxed{\frac{-M_o L}{6EI}} \quad (0.5\,pt)$$

$0 \le x_2 < \frac{L}{2}$

$$\Sigma M_{x_2} = 0, \quad M_o - M = 0$$

$$M = M_o \quad (1\,pt)$$

$$\frac{d^2 v_2}{dx_2^2} = \frac{1}{EI}(M_o)$$

$$\frac{dv_2}{dx_2} = \frac{1}{EI}\left(M_o x_2 + C_3\right)$$

$$v_2 = \frac{1}{EI}\left(\frac{M_o x_2^2}{2} + C_3 x_2 + C_4\right)$$

$@\ x_1 = L\ \&\ x_2 = \frac{L}{2},\quad \frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$

$$\frac{M_o}{6EIL}\left(3L^2 - L^2\right) = -\frac{1}{EI}\left(M_o \frac{L}{2} + C_3\right)$$

$$\frac{2 M_o L^{\cancel{2}}}{\underset{3}{\cancel{6L}}} = -M_o \frac{L}{2} - C_3$$

$$C_3 = -\frac{M_o L}{2} - \frac{M_o L}{3} = -\frac{5 M_o L}{6} \quad (1\,pt)$$

$@\ x_2 = \frac{L}{2},\ v_2 = 0$

$$0 = \frac{1}{EI}\left(\frac{M_o L^2}{8} - \frac{5 M_o L^2}{12} + C_4\right)$$

$$C_4 = -\frac{M_o L^2}{8} + \frac{5 M_o L^2}{12} = \frac{7 M_o L^2}{24} \quad (1\,pt)$$

$$v_2 = \frac{1}{EI}\left(\frac{M_o x_2^2}{2} - \frac{5 M_o L}{6} x_2 + \frac{7 M_o L^2}{24}\right)$$

$$= \boxed{\frac{M_o}{24EI}\left(12 x_2^2 - 20 L x_2 + 7L^2\right)} \quad (1\,pt)$$

$@\ C,\ x_2 = 0$

$$v_2 = \frac{M_o}{24EI}\left(12(0)^2 - 20L(0) + 7L^2\right)$$

$$= \boxed{\frac{7 M_o L^2}{24EI}} \quad (0.5\,pt)$$