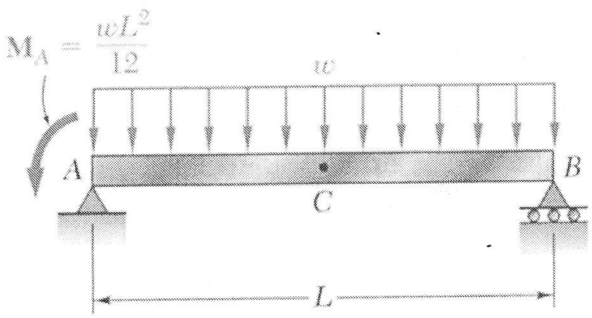
CME 260 – Mechanics of Materials
Exam #7 (04/17/2024)

StarID or TechID (<u>no names</u>) _Grading___________

Do one of the two problems shown below (the second problem is on the back).
<u>Show your work</u> (you will not receive any credit if all you have is a final answer, right or wrong).

1. For the beam below, determine the deflection at C. C is at the center of the beam (a distance of L/2 from both A and B). Your answer needs to be in terms of w, L, E, and I. Simplify your answer by using a common denominator, if applicable.



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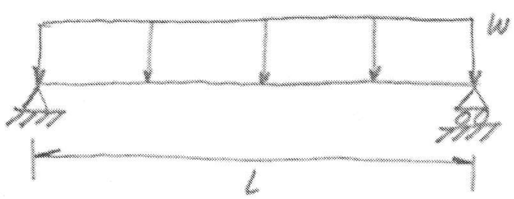
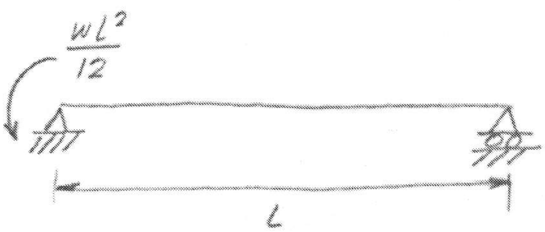


$$@C \rightarrow v_1 = v_{max} = \frac{-5wL^4}{384EI} \quad (3 \text{ pts})$$

+



$$@C \rightarrow x = \frac{L}{2},\ v_2 = \frac{M_0 x}{6EIL}(L^2 - x^2)$$

$$= \frac{wL^2}{12}\,\frac{L}{2(6EIL)}\left(L^2 - \left(\frac{L}{2}\right)^2\right)$$

$$= \frac{wL^{\cancel{3}\,2}}{\cancel{144}_{48}\,EI\cancel{L}}\left(\frac{\cancel{3}}{4}L^2\right)$$

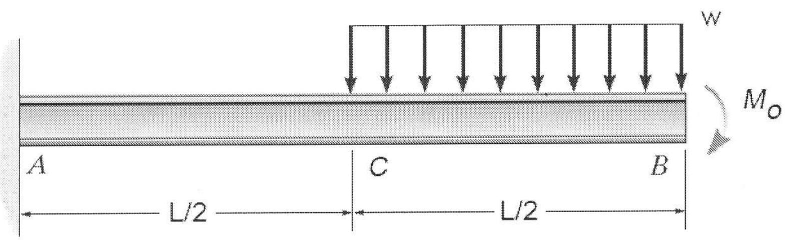
$$= \frac{wL^4}{192EI} \quad (5 \text{ pts})$$

$$v_{@c} = v_1 + v_2$$

$$v_{@c} = \frac{-5wL^4}{384EI} + \frac{wL^4}{192EI} \quad (1 \text{ pt})$$

$$= \frac{-\cancel{3}wL^4}{\cancel{384}_{128}\,EI} = \boxed{\frac{-wL^4}{128EI}} \quad (1 \text{ pt})$$

2. For the beam below, determine the deflection at B. Assume $M_o = wL^2/12$. Your answer needs to be in terms of w, L, E, and I. Simplify your answer by using a common denominator, if applicable.



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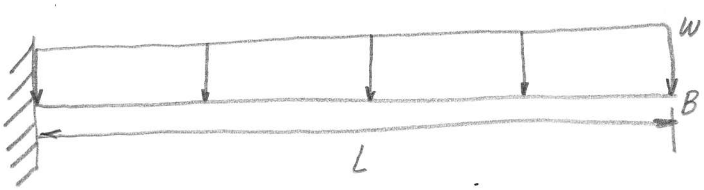
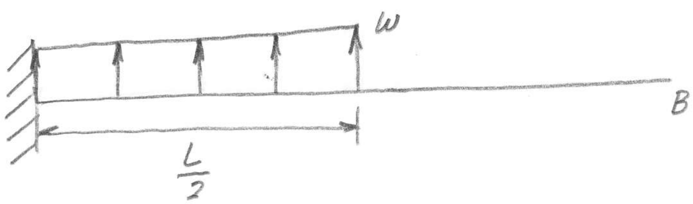


$$V_{1B} = V_{1max} = -\frac{wL^4}{8EI} \quad \text{(2 pts)}$$

+



$$V_{2B} = V_{2max} = \frac{7wL^4}{384\,EI} \quad \text{(3 pts)}$$

+

$$M_0 = \frac{wL^2}{12} \qquad V_{3B} = V_{3max} = -\frac{M_0L^2}{2EI} = \frac{-wL^4}{24\,EI} \quad \text{(3 pts)}$$

$$V_{total} = V_{1B} + V_{2B} + V_{3B}$$

$$= -\frac{wL^4}{8EI} + \frac{7wL^4}{384\,EI} - \frac{wL^4}{24\,EI} \quad \text{(1 pt)}$$

$$= -\frac{48\,wL^4}{384\,EI} + \frac{7wL^4}{384\,EI} - \frac{16\,wL^4}{384\,EI}$$

$$= \boxed{\frac{-57\,wL^4}{384\,EI}} \quad \text{(1 pt)}$$

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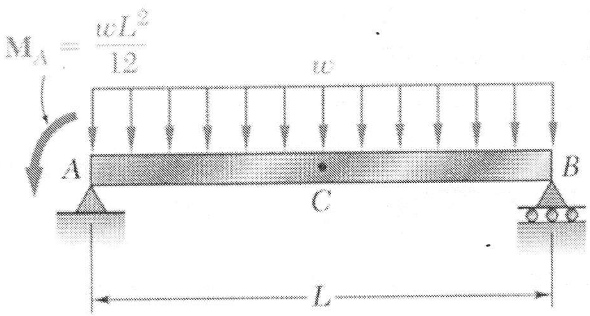
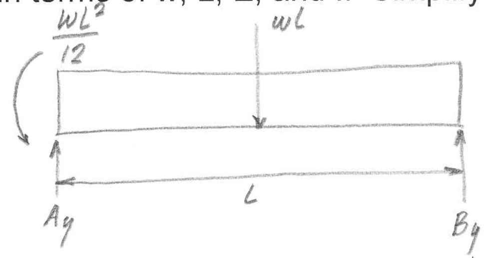
1. For the beam below, determine the deflection at C. C is at the center of the beam (a distance of L/2 from both A and B). Your answer needs to be in terms of w, L, E, and I. Simplify your answer by using a common denominator, if applicable.

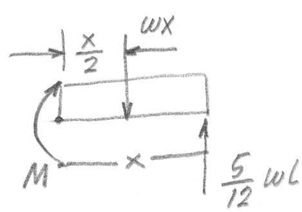




$\Sigma M_A = 0, \quad \frac{wL^2}{12} - wL\left(\frac{L}{2}\right) + B_y L = 0$

$B_y = \frac{wL}{2} - \frac{wL}{12} = \frac{5wL}{12}$

$\Sigma F_y = 0, \quad A_y = \frac{7}{12} wL$



$\Sigma M_x = 0$

$-M - \frac{wx^2}{2} + \frac{5}{12}wLx = 0$

$M = -\frac{wx^2}{2} + \frac{5}{12}wLx$

$\frac{d^2v}{dx^2} = \frac{1}{EI}\left(-\frac{wx^2}{2} + \frac{5wLx}{12}\right)$

$= \frac{w}{12EI}\left(-6x^2 + 5Lx\right)$

$\frac{dv}{dx} = \frac{w}{12EI}\left(-2x^3 + \frac{5}{2}Lx^2 + C_1\right)$

$v = \frac{w}{12EI}\left(\frac{-x^4}{2} + \frac{5}{6}Lx^3 + C_1 x + C_2\right)$

@ $x = 0, v = 0 \longrightarrow C_2 = 0$

@ $x = L, v = 0 \longrightarrow 0 = \frac{-L^4}{2} + \frac{5}{6}L^4 + C_1 L$

$C_1 = \frac{L^3}{2} - \frac{5}{6}L^3 = -\frac{L^3}{3}$

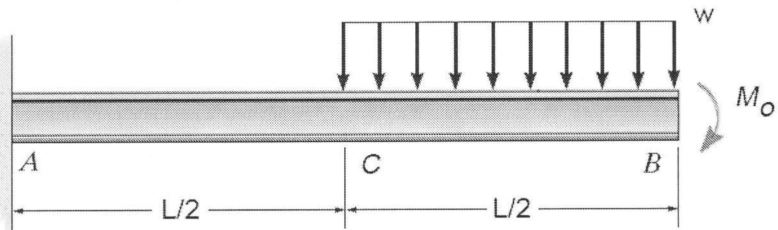
$v = \frac{w}{12EI}\left(-\frac{x^4}{2} + \frac{5}{6}Lx^3 - \frac{L^3}{3}x\right)$

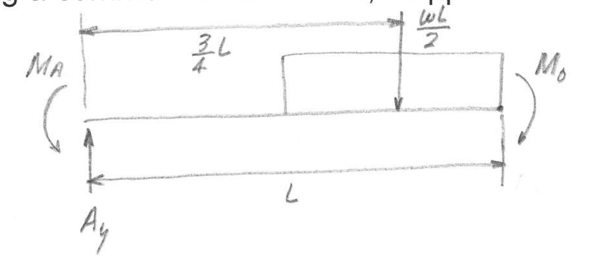
@ $x = \frac{L}{2}, \quad v = \frac{w}{12EI}\left(-\frac{L^4}{32} + \frac{5L^4}{48} - \frac{L^4}{6}\right)$

$= \frac{w}{12EI}\left(-\frac{3L^4}{96} + \frac{10L^4}{96} - \frac{16L^4}{96}\right)$

$= \frac{w}{\cancel{12}_{4} EI}\left(\frac{-\cancel{9}^{3}L^4}{\cancel{96}_{32}}\right) = \boxed{\frac{-wL^4}{128EI}}$

2. For the beam below, determine the deflection at B. Assume $M_o = wL^2/12$. Your answer needs to be in terms of w, L, E, and I. Simplify your answer by using a common denominator, if applicable.





$\Sigma F_y = 0, \quad A_y = \frac{wL}{2}$

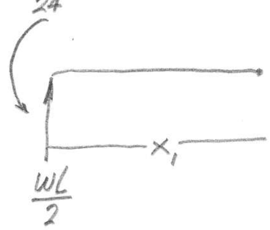
$\Sigma M_A = 0, \quad M_A - \left(\frac{wL}{2}\right)\left(\frac{3L}{4}\right) - M_o = 0$

$M_A = \frac{3}{8}wL^2 + M_o$

subst $M_o = \frac{wL^2}{12}$

$M_A = \frac{3}{8}wL^2 + \frac{wL^2}{12} = \frac{11}{24}wL^2$

$0 \le x_1 \le \frac{L}{2}$ (from left end)



$\Sigma M_x = 0,$

$\frac{11}{24}wL^2 - \frac{wL}{2}(x) + M_1 = 0$

$M_1 = \frac{wL}{2}(x_1) - \frac{11}{24}wL^2$

$= \frac{wL}{24}(12x_1 - 11L)$

$\frac{d^2v_1}{dx_1^2} = \frac{wL}{24EI}(12x_1 - 11L)$

$\frac{dv_1}{dx_1} = \frac{wL}{24EI}(6x_1^2 - 11Lx_1 + c_1)$

$v_1 = \frac{wL}{24EI}\left(2x_1^3 - \frac{11}{2}Lx_1^2 + c_1x + c_2\right)$

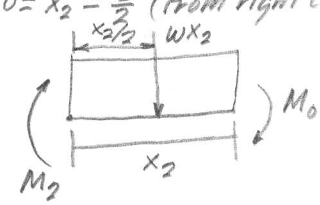
@ $x_1 = 0, \ \frac{dv_1}{dx_1} = 0 \longrightarrow c_1 = 0$

@ $x_1 = 0, \ v_1 = 0 \longrightarrow c_2 = 0$

$\frac{dv_1}{dx_1} = \frac{wL}{24EI}(6x_1^2 - 11Lx_1)$

$v_1 = \frac{wL}{24EI}\left(2x_1^3 - \frac{11}{2}Lx_1^2\right)$

$0 \le x_2 \le \frac{L}{2}$ (from right end)



$\Sigma M_{x_2} = 0, \ -M_2 - \frac{wx_2^2}{2} - M_o = 0$

$M_2 = -\frac{wx_2^2}{2} - \frac{wL^2}{12}$

$= -\frac{w}{12}(6x_2^2 + L^2)$

$\frac{d^2v_2}{dx_2^2} = \frac{-w}{12EI}(6x_2^2 + L^2)$

$\frac{dv_2}{dx_2} = \frac{-w}{12EI}(2x_2^3 + L^2x_2 + c_3)$

$v_2 = \frac{-w}{12EI}\left(\frac{x_2^4}{2} + \frac{L^2x_2^2}{2} + c_3x_2 + c_4\right)$

@ $x_1 = x_2 = \frac{L}{2}, \ \frac{dv_1}{dx_1} = \frac{-dv_2}{dx_2}$

$\frac{wL}{24EI}\left(6\frac{L^2}{4} - \frac{11L^2}{2}\right) = -\left(\frac{-w}{12EI}\right)\left(\frac{2L^3}{8} + \frac{L^3}{2} + c_3\right)$

$\frac{wL}{24EI}(-4L^2) = \frac{w}{12EI}\left(\frac{3}{4}L^3 + c_3\right)$

$\left(\frac{\cancel{12EI}}{\cancel{w}}\right)\left(\frac{\cancel{w}L}{\cancel{24EI}_{2}}\right)(-4L^2) = \frac{3}{4}L^3 + c_3$

$c_3 = -2L^3 - \frac{3}{4}L^3 = \frac{-11}{4}L^3$

@ $x_1 = x_2 = \frac{L}{2}, \ v_1 = v_2$

$\frac{wL}{24EI}\left(\frac{2L^3}{8} - \frac{11}{2}L\frac{L^2}{4}\right) = -\frac{w}{12EI}\left(\frac{L^4}{32} + \frac{L^2}{2}\frac{L^2}{4} - \frac{11L^3}{4}\frac{L}{2} + c_4\right)$

$\left(-\frac{\cancel{12EI}}{\cancel{w}}\right)\left(\frac{\cancel{w}L}{\cancel{24EI}_{2}}\right)\left(\frac{-9}{8}L^3\right) = \left(\frac{L^4}{32} + \frac{4L^4}{32} - \frac{44L^4}{32} + c_4\right)$

$\frac{9L^4}{16} = -\frac{39L^4}{32} + c_4$

$c_4 = \frac{18}{32}L^4 + \frac{39}{32}L^4 = \frac{57}{32}L^4$

$v_2 = \frac{-w}{12EI}\left(\frac{x_2^4}{2} + \frac{L^2x_2^2}{2} - \frac{11}{4}L^3x_2 + \frac{57}{32}L^4\right)$

@ $x_2 = 0, \ v_2 = \frac{-w}{12EI}\left(\frac{57}{32}L^4\right) = \boxed{\frac{-57wL^4}{384EI}}$